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The Equity Portfolio Selection at New York Stock Exchange

Výběr akciového portfolia na New York Stock Exchange

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1. Introduction
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3. Mathematical Formulation of Selected Problems
4. Equity Portfolio Selection using US Dataset
5. Conclusion

Bibliography
List of Abbreviations
Declaration of Utilisation of Results from the Diploma Thesis
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Annexes

References:

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The Declaration

Herewith I declare that I elaborated the entire thesis, including all annexes, independently.

Ostrava dated 20.04.2017

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Contents

1. Introduction

The selection of optimal portfolio is closely related to our life. As the old saying goes, do not put all your eggs in one basket. Hence, we should develop the efficient composition of stock portfolios. Firstly, there are two core issues need to be solved, that is, how to trade off return and risks. Under this background, Markowitz model and a series of developed alternative models come into being. To effectively avoid the risk, ensure the stability of the expected benefit and maximize the investment return, we manage our portfolios by applying these different models. Then, through setting different investment objectives and constraints, investors can determine the optimal portfolio at different risk levels, which denotes by efficient frontier. With the range of efficient frontiers, investors will be able to choose the most satisfying combination in dependence on their preference for risk.

The goal of the thesis is the selection and determination of stock portfolios from trading off return and risk for assets allocation. The analyzed year is the economic recession period to economic recovery period in US from 2007 to 2016. Then, we analyze the different alternative strategies by making comparison about performance and select the appropriate stock portfolio strategy by using the actual data in financial market.

In chapter 2, we will describe the basic information about financial markets, investment decision making and procedures for portfolio management. For investors, the investment steps are significant for the establishment of portfolios. In general, there are three steps in the portfolio investment process, they respectively are planning step, execution step and feedback step. Furthermore, Dow Jones index will be introduced as the benchmark, due to it is the significant index and can be considered as the representative of New York Stock Exchange.

We will introduce the theoretical knowledge about several alternative models and their calculation methods in chapter 3. In the thesis, four strategies are presented, they respectively are Markowitz model, Black's model and Mean-Value at Risk models with different significance level. All of the mathematical formulations will be presented as well, including the setting of objective function and constraints. At the end of chapter 3, we will introduce the measurements of performance.

Chapter 4 is the application part of thesis. In order to make analysis, we firstly need the data description, 23 companies monthly adjust close price are chosen during the last ten years. Furthermore, Dow Jones Index is the main basis for the selection of companies, we choose 23 stocks that not only list in New York Stock Exchange, but also are components stock of Dow Jones index. As the benchmark index, DJIA can reflect the performance of American market and are high capitalization in New York Stock Exchange. Then, we apply the actual data into different alternative models, through the setting of different objective functions and constraints, we can obtain the efficient frontiers and optimal composition for assets allocation. Finally, we make comparison among these alternative strategies and make the suggestions for the selection of appropriate stock portfolio strategy.

In chapter 5, we summarize our thesis and get the conclusion that Markowitz model tends to be safer and more profitable compared with other models.

2. Main Principles of Equity Portfolio Selection

Stock investment management is an important part of asset management, due to the intricate market environment, in order to effectively avoid the risk, to maintain the stability of the expected benefit and maximize the investment return, it is necessary for us to construct equity portfolio strategy rather than investing in single assets. Hence, the main principles and important factors that are closely related to equity portfolios are introduced in chapter 2, such as, financial market and stock market, investment decision making, portfolio management.

2.1 Financial System and Financial Market

Financial markets are composed by markets and a variety of financial intermediaries which can transfer financial assets and risks from one position to another. In this chapter, most of the descriptions are based on the information from CHISHOLM (2007).

There are six reasons why people use financial system, which are introduced by CFA institution[1]: 1）*To save money for future.* 2）*To borrow money for current use.* 3）*To raise equity capital.* 4）*To manage risks.* 5）*To exchange assets for immediate and future deliveries.* 6）*To trade on information.*

In general, financial market refers to the supply – demand relationship and mechanism which are formed by trading financial assets. That means it is a place for conducting financial transaction, and reflects the relationship between suppliers and demanders for financial assets, revealing the concentration and transfer process of capital.

Financial assets themselves are not representative of social wealth, there is no direct contribution to the social-economic, but indirectly play an important role. Financial assets make a separation of right of management from the right of ownership. Holders of financial assets are entitled to share these benefits when the company's physical assets generate income. Thus, financial assets are created on the basis of bond or equity relationships, the holder has the right to claim future cash flow.

[1] CFA institution (2016, p7)

Financial markets create a value discovery process in which the price determined by the interaction of buyers and sellers of financial products in financial markets, people are free to buy and sell in the market which greatly reduces the search costs and information costs for financial product transaction.

2.1.1 Classification of Financial Market

Financial market mainly includes two forms with tangible or intangible. There are different classification from different perspectives.

1) According to the subject matter

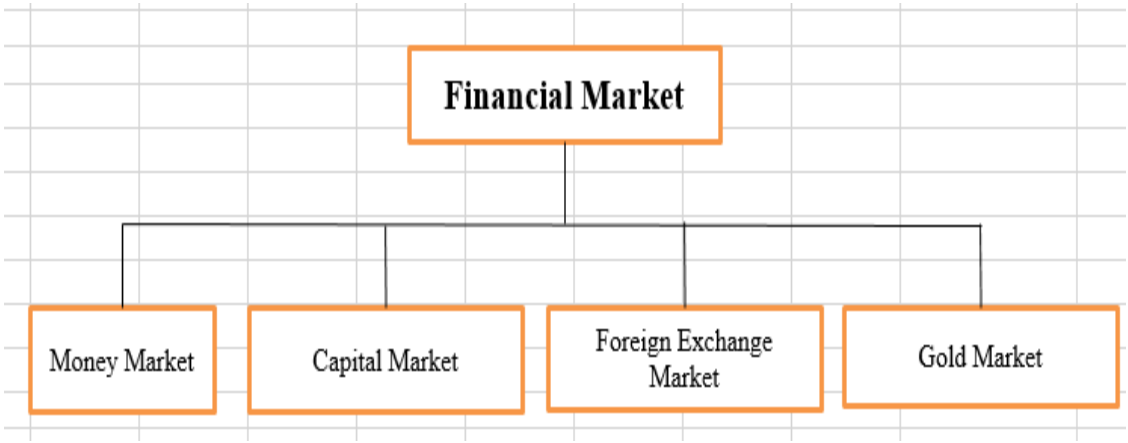


Figure 2.1 Classification of financial market according to different subjects

Money market is the financial market for short-term debt instrument.

Capital market is the financial market for equity instruments, and for debt instrument with a maturity longer than one year.

FX market Refers to the business of foreign currency and foreign currency denominated notes and other securities trading market.

Gold market is trading places that focused on the trading and exchange of gold.

2) According to the issuance and circulation

The primary capital market (issue market) is the financial market that deals with the newly issued securities. The companies, public sectors and governments can raise funds from the issuance of new stocks and bonds in the primary market. This process is usually done by the broker's syndicate (corporate co-organization) or the securities underwriter (who guarantees the purchase of all issued shares or bonds).

The initial public offering (IPO) is the first public issue of financial instrument. The additional equity or debt instrument issued by an already publicly traded firm also belong to the primary market securities.

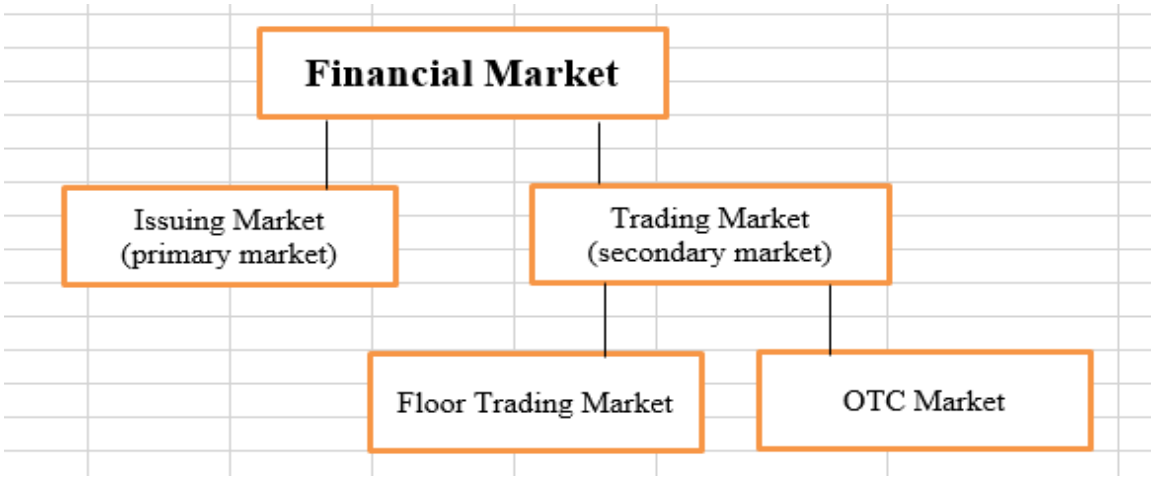


Figure 2.2 Classification of financial market according to issuance

Once the stocks or bonds or other financial instruments are issued in primary market, then they can be traded in secondary market, which means the rebought and resold. In other words, secondary market provides liquidity to participants.

2.1.2 Stock Market

Stocks is equity instrument and is one of securities. It means the ownership of a corporation represented by a stock that is a claim on the corporation's earnings and assets. A firm that issue stocks by selling the part of itself, therefore, the buyer who buys the stock of the company will become a part of owner. A stockholder owns a percentage interest in a firm.

The markets in which stocks are issued and traded is called stock market. The secondary markets for corporate stocks are the most closely watched and reported of all financial security market. This is because stock markets movements are seen as predictors of economic activity.

If a company with higher stock value, it will be easier for them to raise their funds than other companies. And the stock price is fluctuated between fundamental values.

A stockholder in stock market can earn money from two ways, first is the stock price rise, another is the stock dividends paid by the company. If the stock price rise, stockholder can get profit from the difference between selling price and bid price.

There are two types of stock: common stock and preferred stock. And we can find four main distinctions between these two forms of stock: 1) Preferred stock have priority in the distribution of earning. 2) The common shareholders have the company's operating rights, and preferred shareholders generally do not enjoy the company's business participation. 2) The dividend of preferred stock will be paid out before common stocks, generally, the stockholders of preferred stock do not have voting right. 3) The dividend for preferred stock usually is fixed, while the common stock's dividends will be changed depend on the company's profitability.

Dividends can be paid in three forms: cash, services or some benefits and stocks. To avoid the double taxation of dividends, some company choose reinvesting to replace paying dividends.

2.1.3 Stock Exchange

Stock Exchange is a market for the sale of securities, corporate bonds, bonds and other securities. The world's major stock exchanges include the Paris Stock Exchange, the New York Stock Exchange, the Tokyo Stock Exchange, and the Shanghai Stock Exchange and so on.

In our thesis, we mainly consider the New York Stock Exchange.

There are about 2,800 companies listed on the New York Stock Exchange (NYSE), with global market value of 15 trillion U.S. dollars. By July 2004, 30 companies in the Dow Jones Industrial Average were listed on the NYSE except for Intel and Microsoft.

The condition for listing in NYSE is strict. Listing requirements for U.S. domestic companies, such as: 1) the company's previous year's pre-tax profit should not less than 2.5 million U.S. dollars; 2) there are at least 2,000 investors, and at least 100 shares of the stock should be held by each investor; 3) with tangible assets of more than 40 million U.S. dollars. In addition to above requirements, there are some other requirements for non-U.S. companies: 1) Using the accounting principles that generally accepted by the United States. 2) A detailed description of the relative stability of the industry that the company is dependent, the company's position in the industry, and the market situation of company's products.

2.2 Description of Investment Decision-Making

Investment decision-making refers to the process of selection from a number of feasible investment programs. When investors decide to invest their funds into financial markets they firstly need to set their expected investment objectives and assess their risk tolerance.

Considering a specific case: assume there is 1000 USD, one situation is you have a chance to get the certain amount of 1000; the another situation is you have 50% probability to obtain 2000 USD or 50% probability to get nothing. How would you choose? It is difficult to say which one is better, due to there are different risk attitudes for different investors.

For risk averse investors, they would like to choose the situation with a certain amount of 1000 USD. It measures the willingness of people to pay for the risks they faced. In the process of trading off return and risk, they prefer to choose the lower return with lower risk. For risk lover investors, they usually prefer to choose 50% probability to obtain 2000 USD or 50% probability to get nothing, they expect more returns rather than the degree of risk. For investors who are risk neutral, there is no preference about risk and return, they do not avoid or pursue risk, the only criterion for their choice of assets is the size of the expected earnings

Furthermore, it is necessary for us to distinguish investment and speculation.

Investment refers to the economic behavior of a particular economic entity to obtain gains or capital appreciation in the foreseeable future. While speculation is a “zero sum” game, which means the gains of one investor comes from the loss of another investor.

2.2.1 Influence Factors about the Selection of Stock

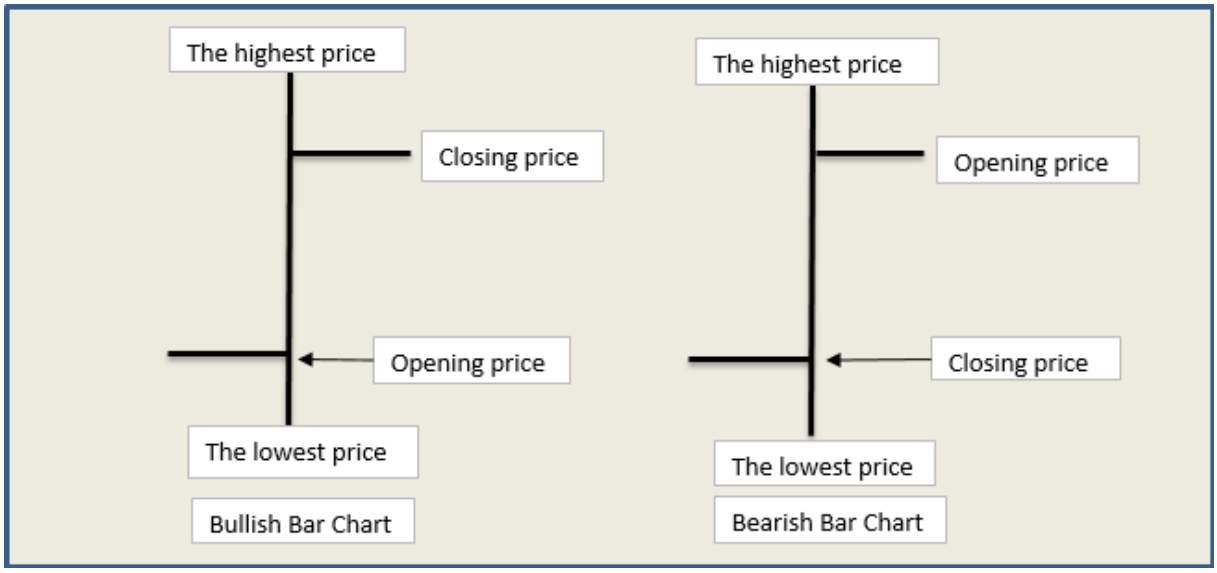
The effective selection of stock is the foundation for the construction of portfolios. Hence, we need to understand what factors can influence the selection of stocks. Here, we mainly introduce two factors that are closely related to the selection, they respectively are technical factors and fundamental factors.

Investors who use technical factors to select stocks, indicates that, they are only interested in security past trading data and the information about future movement tendency of stocks. We can divide technical analysis instruments into two parts, the first is charts and chart patterns;

the second is technical indicators. In this thesis, bar chart and Bollinger bonds will be introduced as the example of instruments.

Bar chart is one type of technical charts. The top point in vertical line represents the highest price during a trading day, while the lowest price is shown by the bottom point. Figure 2.3 displays the difference between bullish bar chart and bearish bar chart, hence, the trading action can be summarized by each bar chart.

Figure 2.3 Bullish bar chart and bearish bar chart



As can be seen from figure 2.3, four main points are included in each bar chart, with trading range between the highest price and lowest price in each trading day. If the price close higher, it indicates there is a bullish bar chart, and we may get profit from trading on this day. If the opening price higher than closing price, which means it is a bearish bar chart, therefore, we may suffer loss from trading on this day.

Technical indicator reveals the calculation results between price and volume, hence, the results can be used to estimates the change of future price. Therefore, Bollinger bonds are created by John Bollinger who combines the concept of moving average and standard deviation in 1980.

Bollinger bands consist of three lines, the middle line is regarded as single moving average line; the upper band represents pressure line of stock price, it is equal to the moving average plus the evenly spaced standard deviation; the lower band stands for the support line, which is equal to the value of the moving averaging minus evenly spaced standard deviation.

Fundamental factors are totally different form technical factors, investors who use fundamental factors to analyze and estimate the intrinsic value of a specific company. The general form of fundamental analysis is represented by ratios.

Price to earnings ratio is calculated by the stock price divide by the annual earnings per share. It also can be considered as a multiple, as it indicates the willing of investor to pay for 1 dollar of earnings. Such as, if the price to earnings ratio is 10, it represents 10 USD are willing to pay by investor for the earning of 1 USD.

Price to book value ratio will be used for companies with low returns. It is the ratio of stock's capitalization to book value, therefore, it reflects the ratio of asset profitability to initial cost. Generally, the lower price to book value ratio indicates a good opportunity for investment.

In recent years, many evaluators have begun to use the price / sales ratio, due to the advantages compared with P/E, P/BV: as we know, P/E or P/BV becomes meaningless if the value is negative, however, P/S ratio can be used in anytime and meaningful even for the most difficult company. Usually, the smaller the market rate (for example, less than 1), the higher the investment value is considered.

2.2.2 Dow Jones Index

In our thesis, we choose Dow Jones Index as the benchmark index. As we know, The Dow Jones index is one of the world oldest and the world's most important stock indices, it can reflect the performance of American economy. The Dow Jones index was developed in 1884, founder named Charles Henry Dow, it is an arithmetic average price index.

Figure 2.4 Evolution of Dow Jones Index

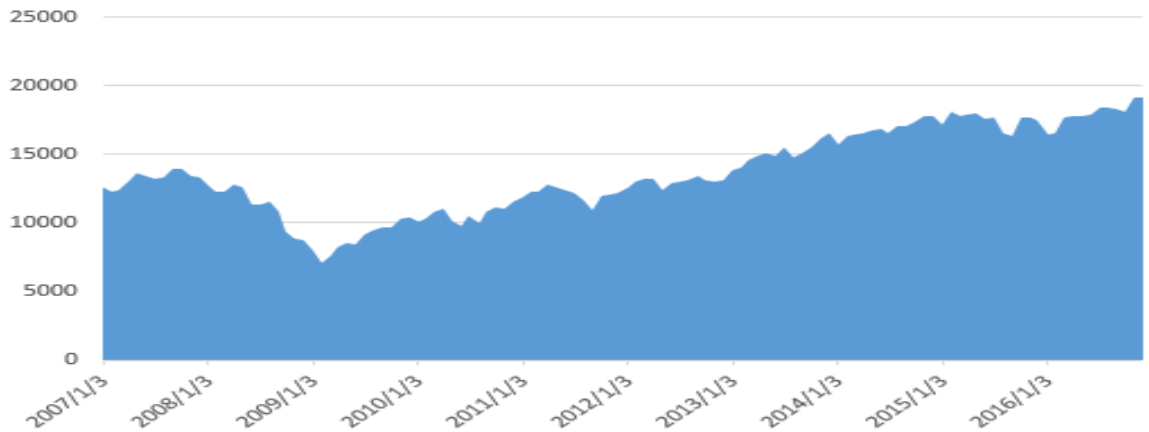


Figure 2.4 indicates the evolution of DJIA, we can find the whole tendency is positive from 2007 to 2016. Beside 2009, due to the negative influence of financial crisis. As we state before, DJIA can be regarded as the representative of New York Stock Exchange, hence, we choose Dow Jones Index as the basis for the selection of stocks. Although there are 30 component stocks in DJIA, we only choose 23 stocks to analysis. These 23 stocks are listed in New York Stock Exchange and can apply enough ten years historical data.

2.3 Portfolio Management

In subchapter 2.3, we will mainly introduce the importance of portfolio approach to achieving the financial goal for all types of investors. We compare individual and institutional investors' different financial needs. And then we describe the process of portfolio management. Through compare and contrast investment management products such as mutual funds and pooled investment, we can apply them to portfolio approach. The description is mainly based on MADURA (2008).

For a portfolio perspective on investing, there are a number of challenges faced by individuals and institutions, and how to decide the investment for future needs is one of the biggest challenges. For institutions such as insurance companies, the goal might be raise funds for the future liabilities because of insurance claims. In terms of individual point of view, the goal might be raise funds for the life needs of retirement. Hence, whether it is institution or individual, they should focus on how to invest to achieving their different goals. But there exist one important question for investors: When we invest, do we invest in a single securities or a portfolio?

In this section, we will talk about the advantages why investors invest in portfolio rather than single securities.

There are several reasons why investing in portfolio is important:[2] *1) Portfolio diversification. 2) Reduce risk. 3) Composition matters for risk – return tradeoff. 4) Not necessary downside protection. 5) The emergence of modern portfolio theory*

[2] Michael G and Jerald E (2008, p69)

Portfolio diversification

Diversification can help investors avoid disasters. It means investors should spread the funds into different industries, businesses, markets, etc., in order to avoid investment risk, and achieve the pursuit of maximizing investment profit.

We will illustrate several types of portfolio diversification:

1) The diversification of industry types.

Dispersed industry category means that when securities investors purchase stocks, they should not focus on putting their funds to buy shares of the same industry. This is because, in the event of whole industry downturn, then the stock price in the same industry will fall sharply, if investor invest in one industry, they can't receive the expected investment return, and even the principal will be lost.

2) Diversification of business units

Decentralization of business units means that securities investors do not use all the funds to buy one stock, they should invest in a number of enterprises. This diversification can help us avoid the risk of investment loss due to poor management of business.

3) Diversification of areas

Due to the difference of local laws and regulations or various local business environment, so that the economic impact of regional industries, enterprises operating is different. Hence, investors can invest in different places to reduce risk, and increase opportunities to gain more profit.

4) Decentralized time

Because of industries own characteristics, there will be production cycle and exist slack season and busy season distinction. In the period of enterprise cut off and off-season, the stock price will certainly be affected and fell. In addition, generally the share price will be higher before dividend payout, so investors should realize the dividend payout time of various stock. Therefore, investors should invest their money at different times to avoid risks.

5) Diversification of plate

Some countries' securities market has a plate effect, securities investors should also focus on the characteristics of two or more different plate to avoid the plate risk.

6) Decentralized market

As the ups and downs of different markets will not occur at the same time, investors can diversify their funds into different markets. Take stock as an example, the factors that affect the stock price including international and local interest rates, local and international economic development cycle and the local government's fiscal policy and monetary policy. The unequal factors between these markets have reduced the correlation coefficient of each other's share price, which explains why the volatility of the international stock portfolio is lower than that of a single country.

Reduce risk

In general, portfolio can provide equivalent expected return with lower volatility compared with single security. As we know, the volatility of return can be expressed as a measurement of standard deviation which stands for risk.

There was a lack of specialized research program based on the concept of portfolio diversification, so this is where TOBAM's own research program is rooted. What do we see the core issue is: how to measure "how much" about the diversification of the portfolio in an investment?

Our research has allowed us to define a diversification by using mathematical measure. The diversification ratio[3] (DR) opens the door to a systematic approach to building a well-diversified portfolio.

The definition of a portfolio's diversification ratio is the ratio of the weighted average volatility of individual securities in that portfolio, divided by the volatility of the overall portfolio, we present it in formula (2.1):

$$DR(p) = \frac{combination\ of\ the\ risks}{Risk\ of\ the\ combination} = \frac{\sigma_1\omega_1 + \sigma_2\omega_2 + \cdots + \sigma_n\omega_n}{\sigma_p} \qquad (2.1)$$

Where σ_1 is each asset's volatility, and σ_p is the volatility of portfolio.

Diversification Ratio (DR) reflects the essence of the diversification of the portfolio:

[3] Source: MACKENZIE Investment. [Online]. [07.2016]. Available on: http://www.mackenzi Investments.com

The numerator of DR which stand for the combination of the assets risk is higher than the denominator of DR which means the risk of the assets combination, provided that the assets are not completely relevant.

Diversification Ratio measures the extent to which a particular portfolio is diversified, so that the ratio can be used to compare the portfolio. If the portfolio is more diversified, it illustrate the higher diversification ratio.

Maximizing the diversification ratio of the portfolio has attractive features and maximizes the number of independent sources of exposure that it exposes. In this sense, a portfolio with a high DR is considered to be biased and does not use any point of view (i.e. market bias) about future risk compensation for constituent stocks. In other words, a truly diversified portfolio does not reflect any speculative view of the market.

In last section, we illustrate that portfolio can help investor avoid disaster compared with investing in a single security. While in this section, we focus on the introduction of risk reduction through portfolio, which is one of the most important advantages of portfolio for investors.

Composition of risk and return

The risk-reward trade-off principle refers to a trade-off between risk and reward. Investors must weigh the return and risk, take greater risks for pursuing higher returns, or accept lower returns for risk reduction.

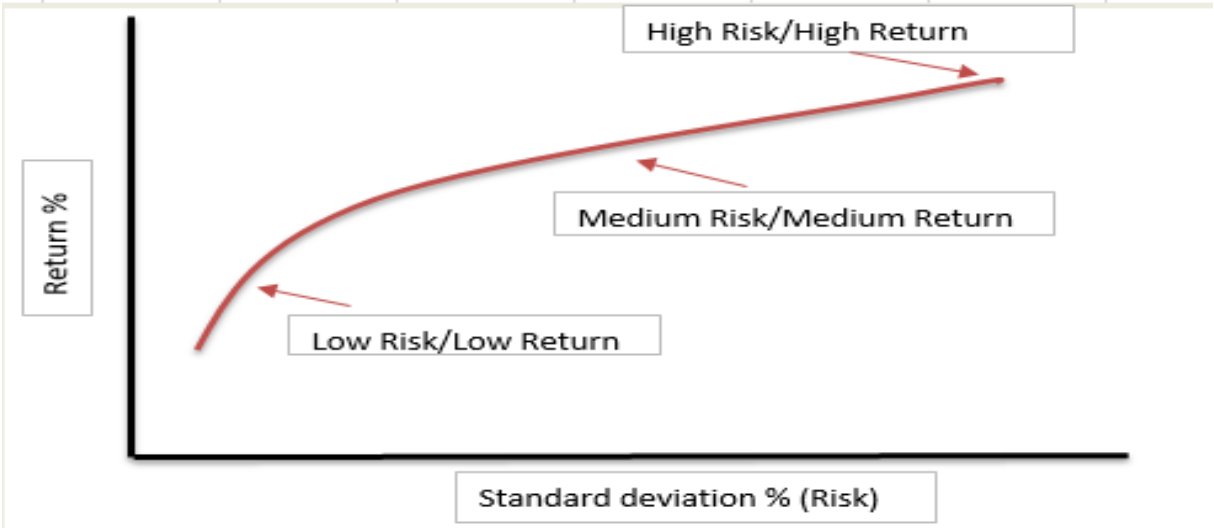
In this section, we research the combination of the set of stocks and trade – off the volatility of shares and expected return for the portfolio. We can see the efficient set of stock portfolios from figure 2.5.

The efficient set was originally developed by Markowitz, developed as a method of portfolio selection. From figure 2.5 we can find the efficient frontier for a portfolio. For a rational investor, they are risk aversion and prefer benefits.

Effective frontier also known as effective portfolio theory, which means that an effective investment must meet one of the following conditions: ① the same risk under the conditions of the greatest benefit; ② the same income under the conditions of the smallest risk. And the effective frontier is the set of portfolio with the same level of return and lower level of risk.

A feasible set is the aggregate of all portfolios formed by risk assets in the capital market. A collection of all portfolio formed by *N* securities, which includes all possible combinations in real life. That is, all possible combinations will be located on or within the bounds of the feasible set.

Figure 2.5 Efficient set of portfolio



As can be seen, the return is expressed by expectations and standard deviation stands for the risk. For a rational investor, they are risk aversion and prefer benefits. Hence, the effective set is a curve that tilts to the upper right, which reflects the principle of "high yield, high risk"

Not necessarily downside protection

The downside risk of a share price is the risk that the future stock price trend may be lower than the expected target price of the analyst or investor. Downside protection is generally used in the sale of a call option, which is a protective pad against the loss, to prevent the underlying stock price declining from selling the call option. It can protect investors from the adverse effects of downside market.

As we already know, the value of some subset in a portfolio will rise while another will go down at the same time. This is a major reason why portfolio can reduce risk effectively.

However, there is a crucial issue that the security's return can be changed through the co-movement and correlation in the portfolio, which are detrimental to investors. Hence, in fact, diversification is ineffective and false if the economy recession contagion and fail in a worldwide.

Modern portfolio theory

Modern Portfolio Theory (MPT) emerged in 1952 and published by Markowitz. It illustrates a rational investor who uses diversified investment to optimize their portfolios. In theory, the reward of an asset is a random variable. The reward should also be a random variable, and the return of the portfolio therefore has an expected value and a variation. As illustrate by CECCHETTI and Kermit (p149), investors should pay attention to the relationship of one individual security in the portfolio to another rather than simply holing portfolios, which is the most important summarization of modern portfolio theory. The study provides a view that the asset risk should be related to the remaining systematic risk and non diversifiable risk.

2.4 Process of Portfolio Management

For investors, the investment steps is important for the establishment of portfolio. There mainly are three steps in the portfolio management process, they are respectively are planning step, execution step and feedback.

2.4.1 The Planning Step

To establish and manage investment portfolio of clients, firstly should have the information about client's needs and write the investment policy statement (IPS). The IPS is a document and provide general rules and state some investment goals, objectives and restraint.

The objective of the portfolio is not only the basis for the construction and adjustment of the securities portfolio, but also the benchmark for the performance of the portfolio management. In general, the objectives of the portfolio include two aspects: one is the revenue target, including the guarantee of the principal safety, and to obtain a certain proportion return of capital and a certain growth rate of capital; the second objective is the risk control, including assets liquidity requirements and the determination of the maximum loss range.

2.4.2 The Execution Step

According to the information of client's IPS, portfolio manager will create an appropriate portfolio. This is the core steps of implementation of the securities portfolio management, and directly determine the combination of benefits and risks.

Asset allocation

Asset allocation is based on the investors' investment objectives, and allocate their investment into different types of assets, determine the weights of various assets in the portfolio, in addition to receive the ideal return, the risk should be minimum.

The allocation of assets is mainly dependent on the following four factors: 1) Investment objectives. If the investor's goal is to maintain or secure a stable income, the main proportion of the portfolio should be the bond; on the contrary, if the investment target is to add value to capital, it should be stocks investment. 2) The affordable risk. If investors are not willing to take risks, they should focus on investing on bonds rather than on stocks. 3) The time limit for the investment plan. Such as investors prefer long-term investment, for example: 20 years, the investment portfolio may wish to make progress, that is, buy more stocks, buy less bonds; the contrary, in the case of short-term investment, then should hold more bonds. 4) Investor's age. Many professional investment managers or consultants also advise investors to consider age factors when they are allocating assets. Such as an 30 years old investors, put 30% of the funds invested in bonds and cash, the remaining 70% is invested in stocks; and so on.

As we know, the economic condition can influence assets' return, so economists establish the top down view in order to forecast economic conditions.[4]

Top down: *A top down analysis begin with the consideration of macroeconomic condition. Analysts assess the industry and market condition with the aim of investing in those that are expected to perform well through the current and forecast economic environment. Finally, specific companies within these industries are considered for investment.*

[4] Michael, Jerald and Wency. Principles of portfolio and Equity Analysis. p.157. ISBN: 978-0-470-91580-6.

Bottom down: *Focus on company's particular circumstance such as business prospects and management quality instead of emphasize industry analysis or economic cycle is bottom down analysis. It is less concerned with broad economic trends than is the case for top down analysis.*

Security analysis

Through security analysis, investors can make an analysis for a number of tradable financial instrument. And the assets' value can be determined in the portfolio based on this analysis. In general, it can be classified into three kinds: fundamental analysis, technical analysis and quantitative analysis. These knowledge help analysts recognize preferred investment.

Portfolio construction

Once the weight of each assets is determined and the security analysis is done, then analyst construct the portfolio based on the client's needs and objective.

In the process of portfolio construction, risk management is very important. Trading also included in construction phase. Therefore, in many investment firms, the trades will be passed to a side of buy trader by portfolio manager, it means, a colleague who specializes in securities trading, he will contact a stockbroker or a dealer to execute the trade. [5]

2.4.3 The Feedback Step

The feedback step help investor rebalance their portfolio due to the change in economic condition. This step includes monitoring and rebalancing of portfolio and performance measurement.

[5] Michael, Jerald and Wency. Principles of portfolio and Equity Analysis. p.159. ISBN: 978-0-470-91580-6.

3. Mathematical Formulation of Selected Problems

In this chapter, we will mainly introduce some important alternative models and their mathematical formulations. We mainly pay attention to Markowitz model, Black's model and Mean-VaR model with different significance levels. To compare these different models, performance measurement will be introduced as well.

3.1 Basic Descriptive Data Formulation

In this subchapter, we focus on the description of basic input data, what is necessary for the calculation of different models. Here, we assume a given $m \times n$ matrix, where *m* represents our historical data's length, and *n* stands for the number of risky assets that we hold. $P_{i,t}$ represents the stock price of asset *i* at time *t*, $P_{i,t-1}$ shows the previous price of asset *i* at time *t-1*.

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{3.1}$$

Where, $R_{i,t}$ express the obtained return at time *t* of the asset *i*, it is equal to the price of asset *i* minus the previous (*t-1*) year's price, then divide to the price of assets *i* at previous year *t-1*.

$$E(R_i) = \frac{1}{m} \cdot \sum_{t=1}^{m} R_{i,t} \tag{3.2}$$

As presented in Equation (3.2), $E(R_i)$ denotes the mean of $R_{i,t}$ return. In fact, we use mean of return as asset *i*'s expected return. As we know, we usually use standard deviation or variance to represents risk. How can we calculate the risk for an assets or portfolio, it is very important for an investor, because risk is closely related to investor's potential gain or loss. Next, the variance of return is expressed by Equation (3.3),

$$\sigma_i^2 = \frac{1}{m} \cdot \sum_{t=1}^{m} \left[R_{i,t} - E(R_i)\right]^2 \tag{3.3}$$

The risk of assets is denoted as variance and standard deviation. Equation (3.3) represents the asset *i*'s variance over *m* period. The risk standard deviation can be seen in Equation (3.4), which stands for root square of variance.

$$\sigma_i = \sqrt{\sigma_i^2} \tag{3.4}$$

Next, covariance can be calculated. The covariance is shown in Equation (3.5), which express the relationship between assets *i* and asset *j*. It measure how much asset *j* can be change on return, due to the return's change on asset *i*. If asset *i* and *j* are positively related, there will be same tendency between these two assets. On the contrary, the assets tendency inversely related as the covariance value is negative.

$$\sigma_{i,j} = \frac{1}{m} \cdot \sum_{t=1}^{m} \left[R_{i,t} - E(R_i)\right] \cdot \left[R_{i,j} - E(R_i)\right] \tag{3.5}$$

The correlation coefficient is a statistical indicator used to reflect the degree of correlation between variables.

$$\rho_{i,j} = \frac{cov(R_i; R_j)}{\sigma(R_i) \cdot \sigma(R_j)} \tag{3.6}$$

Where $\rho_{i,j}$ is the correlation between assets *i* and *j*, for fast calculation, we also can use analysis tool "correlation" with Excel to calculate the correlation matrix.

3.2 Markowitz Model

Investment in securities and other risky assets, we firstly needs to solve two core problems: expected returns and risks. So how to determine the portfolio risk and return and how to balance the two indicators of asset allocation is an urgent problem need to be solved for market investors. That's why in the 50's and early 60's, Markowitz theory came into being. This information and formula mainly based on Zdenek (2004).

The Markowitz model is the type of the mean variance model, it only allowed to invest into risky assets, the riskless investment as well as short selling for risky assets is forbidden. *Markowitz (1952)* illustrate that if we invest in a portfolio with *i* assets and x_i *i*s the weight for each assets, the short selling are eliminated at the same time, *E(R)* stands for expected return and $E(R) = [E(R_1), E(R_2), \ldots E(R_n)]^T$, then we can get $\mathrm{E}(R_p)$ the formula of portfolio's expected return.

$$E(R_p) = \sum_{i=1}^{n} x_i \cdot E(R_i) = x^T \cdot E(R) \tag{3.7}$$

In addition to the expected return $E(R_p)$ of portfolio, we should focus on the risk of portfolio.

$$\sigma_p^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i \cdot \sigma_{i,j} \cdot x_j = x^T \cdot \boldsymbol{C} \cdot x \tag{3.8}$$

Where, C is covariance matrix. In mean – variance model, we treat variance as risk. And the portfolio standard deviation are presented in equation (3.8).

$$\sigma_p = \sqrt{\sigma_p^2} \tag{3.9}$$

There are three distinct steps to solve the problem about the selection of optimal relative composition of efficient portfolio.

First we should find the minimum risk portfolio. The following table 3.1 can be regarded as Equation (3.10).

Table 3.1 Minimum risk portfolio

Objective function:

$$\sigma_p \rightarrow min.$$

Constraints:

$$\sum_i x_i = 1, \qquad (C1)$$

$$x_i \geq 0, \text{for } i = 1,2,\dots N, \qquad (C2)$$

$$Where, \qquad \sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \cdot \sigma_{i,j} \cdot x_j = x^T \cdot \boldsymbol{C} \cdot x.}$$

The constraints C1 means the sum of x_i is equal to 1, and C2 represents the short selling is not exist.

Finding the maximum expected return portfolio is the second step. The following table 3.2 can be regarded as Equation (3.11).

Table 3.2 Maximum expected return portfolio

Objective function:

$$E(R_p) \rightarrow \max.$$

Constraints:

$$\sum_i x_i = 1, \qquad (C1)$$

$$x_i \geq 0, \text{for } i = 1,2,\dots N, \qquad (C2)$$

$$Where, \qquad E(R_p) = \sum_i x_i \cdot E(R_i) = \vec{x}^T \cdot E(\vec{R})$$

Calculating the expected return and the standard deviation of portfolios in equidistant intervals. The following shows Equation (3.12).

Table 3.3 Equidistant point of efficient frontier

Objective function:

$$\sigma_p \rightarrow \min.$$

Constraints:

$$\sum_i x_i = 1, \qquad (C1)$$

$$x_i \geq 0, \text{for } i = 1,2,\dots N, \qquad (C2)$$

$$E(R_p) = E(R_{p-gen}), \qquad (C3)$$

$$Where, \qquad \sigma_p = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} x_i \cdot \sigma_{i,j} \cdot x_j} = x^T \cdot \boldsymbol{C} \cdot x$$

$$Equidistant\ interval = \frac{E(R_{p,max}) - E(R_{p,min})}{119}$$

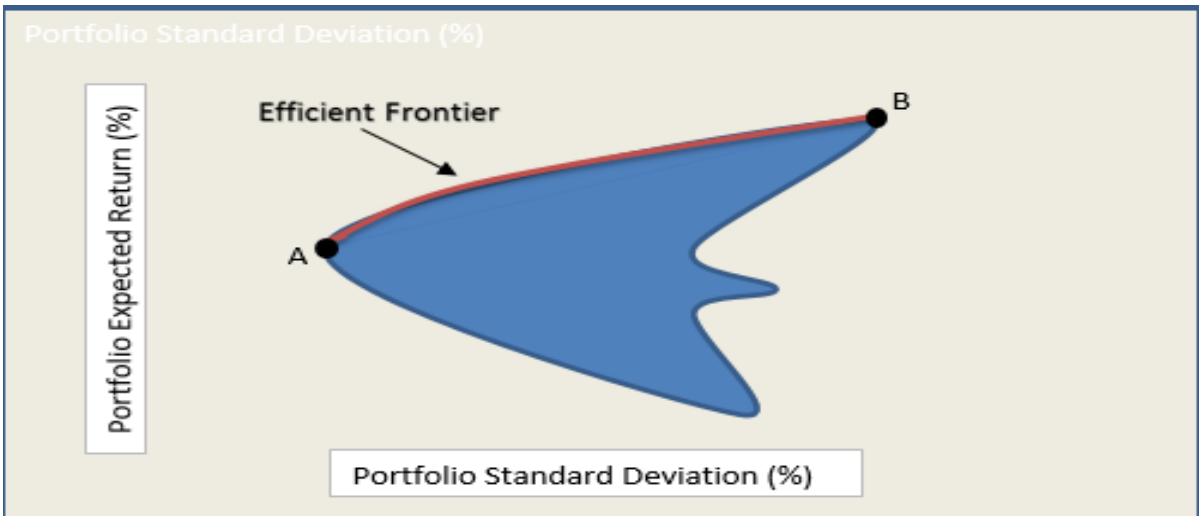
$$E(R_{pj}) = E(R_{p-1}) + equidistant\ interval.$$

Constraints 3 we ensure $E(R_p)$ the expected return will equal to requested mean return $E(R_{p-gen})$ as specified initially for a given equidistant point.

Markowitz's portfolio theory not only reveals the determinants of the asset risk, but also, more importantly, concluded that "the expected return on assets is determined by the size of their own risk," the asset price (Individual assets and portfolio assets) are priced by their risk, therefore, the price of a single asset is determined by its variance or standard deviation, the

price of the portfolio is determined by its covariance. From figure 3.1, we can clearly see the efficient frontier of Markowitz model.

Figure 3.1 Efficient frontier of Markowitz model



As can be seen from figure 3.1, the red line between point A and B is efficient frontier of Markowitz model, it indicates all efficient composition of stock portfolios. However, for investors, they are interested in finding the best optimal portfolio, hence, risk profile of investors is key factors for them to choose the optimal portfolio. If lack the specified investor's risk information, thus only efficient set will be constructed. Depending on investor different level of risk averse, they can select the optimal portfolio to maximum their own profit. Point A is the minimal risk portfolio with minimizing the standard deviation, point B represents the maximum expected return but also with the highest risk.

3.3 Black's Model

Black's model is also one type of mean variance model, but, different from Markowitz model. As we introduced in subsection 3.2, the short selling is forbidden in Markowitz model, which means the weight for each assets should not be negative. Here, in Black's model, it is allowed for investors to invest into assets with short sale. However, only invest in risky assets as well. Short selling means the sale of stocks when investors expect the stock price in future market will be down, they can get profit from differences. It indicates the negative weights of stocks can be allowed in Black's model.

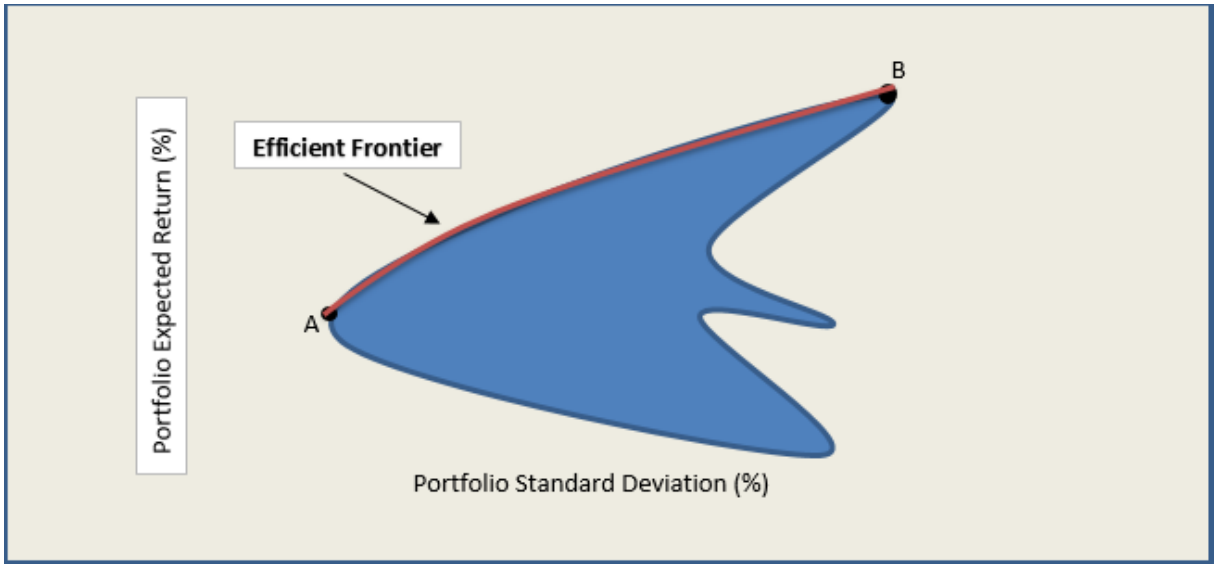
The application of Back's model is very similar with Markowitz model, hence, the objective function is same. The only difference can be found in constraints.

Table 3.4 Change of constraint from Markowitz to Black's model

Change the constraints from Markowitz model $x_i \geq 0, \text{for } i = 1,2, \dots N, \qquad (C2)$ to Black's model $x_i \geq -1, \text{for } i = 1,2, \dots N, \qquad (C2)$

The change of constraints can be denotes as Equation (3.13). As the short selling are allowed in Black's model, hence, the weights of stocks can be positive or negative in Black's model with the permission of short selling. Although the negative weights can be exist, it could not lower than -1. Figure 3.2 displays the efficient frontier of risky assets with short selling.

Figure 3.2 Efficient set with Black's model



Point A stands for the most minimal standard deviation portfolio. The highest point B represents the portfolio whose expected return are maximal. The red line indicates all the efficient portfolio.

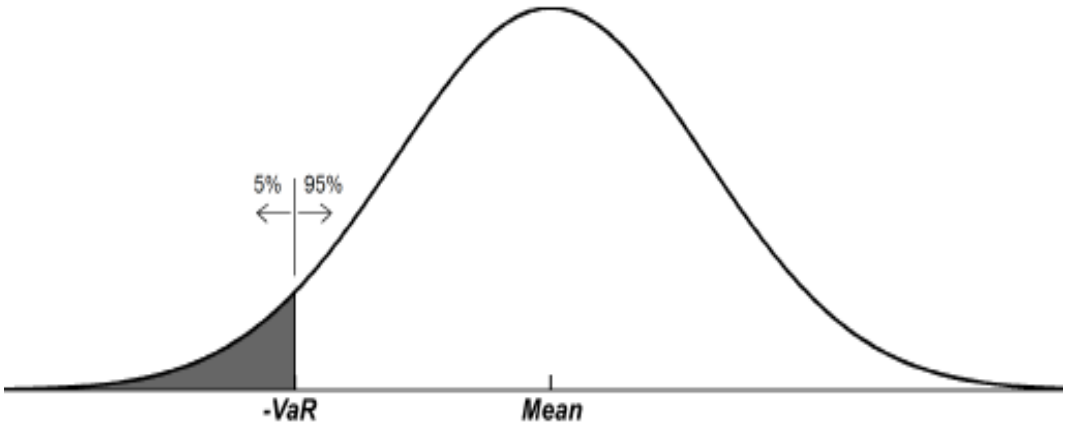
By make comparison between Markowitz model and Black's model, we can find the efficient frontier is much more wider and longer than Markowitz model, it indicates Black's model can provide investors more choice about the selection of optimal portfolio.

3.4 Mean-Value at Risk Model

There is a crucial problem for investment decision making and assets management, that is, how to effectively manage and reduce financial risks. As we know, financial market is instability, there is a lot of risk can be faced by us when we make financial investment. Hence, it is necessary for us to estimates Value at Risk, due to the advantage of transform all risks into a single number is permitted in this method.

Value at risk (VaR) is the maximum loss of a financial asset or security portfolio under the volatility of market. More precisely, it indicates the maximum possible loss at a certain probability level (confidence level) over a specific period of time in the future. Figure 3.3 illustrates the definition of VaR, it indicates at 5% significance level, the maximum loss of specific firms. In general, the choice of confidence intervals reflects, to a certain extent, the different risk preferences of financial institutions. Choosing a larger confidence level means a higher level of risk-aversion, hoping to get a better predictive result, therefore, hoping that the model is more accurate for extreme events. Depending on their respective risk preferences, the choice of confidence level are also different.

Figure 3.3 The definition of Value at Risk



VaR is mainly aimed at risk of financial market, it is calculated in the assumption of a normal distribution, which means that does not consider the situation as a collapse of market. Thus, the VaR measures what is expected to occur during the day-to-day operations of the organization. For the calculation of VaR, following data is required: the price and volatility of

each asset in the portfolio, the correlation of assets as well. In general, it is assumed that the composition of the portfolio is random and subject to a normal distribution.

As the predetermined profit level, calculated by some specific significance level α, should be higher than the random profit of portfolio $\Delta\widetilde{\Pi}$, thus, the measurement of Value at Risk can be states as a negative loss.

$$P_r\left(\Delta\widetilde{\Pi} \leq +PROFIT\right) = \alpha \tag{3.14}$$

As Value at risk (VaR) indicates the possible loss at a certain probability level (confidence level) over a specific period of time in the future. The loss is defined as the level of VaR. Then, we formulate $PROFIT = -VaR$ to express the negative loss of profit, hence, we can get Equation (3.15) from the modification of Equation (3.14).

$$P_r\left(\Delta\widetilde{\Pi} \leq -VaR\right) = -\alpha \tag{3.15}$$

Table 3.5 Mathematical formulation of Value at Risk

Objective function:

$$VaR = -E\left(r_p\right) - \Phi_\alpha^{-1} \cdot \sigma_p \rightarrow min.$$

Constraints:

$$\sum_i x_i = V_0 \qquad for\ i = 1,2,\dots N, \qquad (C1)$$

$$x_i \geq 0.01 \cdot V_0 \qquad for\ i = 1,2,\dots N, \qquad (C2)$$

$$x_i \leq V_0 \qquad for\ i = 1,2,\dots N, \qquad (C3)$$

$$E\left(r_p\right) = \sum_i E(r_i) \cdot x_i = E^T(\vec{r}) \cdot \overrightarrow{x}, \qquad (E1)$$

$$Where, \quad \sigma_P^2 = \vec{x}^T \cdot C \cdot \vec{x} = \sum_i \sum_j x_i \cdot \sigma_{i,j} \cdot x_j \qquad (E2)$$

$$\sigma_{i,j} = \sigma_i \cdot \rho_{i,j} \cdot \sigma_j. \qquad (E3)$$

Table 3.5 is denoted by Equation (3.16). Here, Φ_α^{-1} represents the inverse function for the normal distribution under the specific level of probability α. The covariance and correlation need to be calculated and denotes by $\sigma_{i,j}$ and *C*. The objective function indicates we should minimize the Value at Risk at the particular significance level. Constraint $(C1)$ indicates the

total amount of principle we invest into portfolios, Constraint $(C2)$ and $(C3)$ shows the restriction for our investment.

3.5 Portfolio Risk

This section mainly analyzes the components of portfolio risk. In fact, it describes how to consist the assets in a portfolio to reduce the potential risk without reducing return by the low correlation of assets. Most description of this part is based on CHISHOLM, Andrew M (2007) and MADURA, J (2008).

3.5.1 Portfolio of Two Risky Assets

Diversification means when we are in securities investment, we should distribute our funds into a variety of assets. In order to achieve the purpose of risk diversification, we should choose the assets which the correlation among these assets' expected return is relatively low. Diversification can help investors reduce risks while not damage their expected return.

In order to elaborate the effect of diversification, we consider a case with two financial assets S_1 and S_2:

So the covariance is represented by:

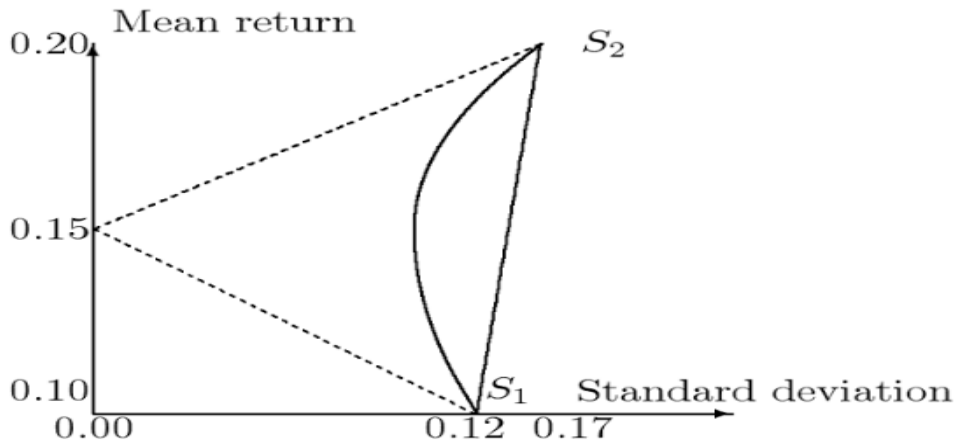
$$\sigma_{12} = Cov(R_1, R_2) = \rho_{12}\sigma_1\sigma_2 \tag{3.17}$$

R_1 and R_2 respectively stands for the return for assets S_1 and S_2. ρ_{12} means the correlation coefficient between financial assets S_1 and S_2. If a portfolio P with a percentage of x, which represents wealth, and then invested x into financial assets S_1, (1-x) invested into S_2. So the return for R_p will be:

$$E[R_p] = xE[R_1] + (1-x)E[R_2] \tag{3.18}$$

$$\sigma_p^2 = x^2\sigma_1^2 + (1-x)^2\sigma_2^2 + 2x(1-x)\sigma_1\sigma_2\rho_{12} \tag{3.19}$$

Figure 3.4 Relation between risk and return



Source: Michael G and Jerald E (2011, p68).

This figure 3.4 indicates the combination of two assets based on the correlation coefficient ρ_{12}.

1) If $\rho_{12} = 1$, it represents the expected return and standard deviation are linear combinations for those assets.

2) If -1 < ρ_{12} < 1, the minimal variance portfolio is solved by following formula:

$$\frac{\partial \sigma^2(R_p)}{\partial x} = x\sigma_1^2 - (1-x)\sigma_2^2 + (1-2x)\sigma_1\sigma_2\rho_{12} = 0 \qquad (3.20)$$

The optimal percentage is given by:

$$x^* = \frac{\sigma_2^2 - \sigma_1\sigma_2\rho_{12}}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}} \qquad (3.21)$$

The variance meet:

$$\sigma^2(R_{p^*}) = \frac{(1-\rho_{12}^2)\sigma_1^2\sigma_2^2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}} \qquad (3.22)$$

If, financial assets S_1 is less risky than S_2:

$$\sigma^2(R_{p^*}) - \sigma_1^2 = -\frac{\sigma_1^2(\sigma_1 - \sigma_2\rho_{12})^2}{\sigma_1^2 + \sigma_2^2 - 2\sigma_1\sigma_2\rho_{12}} \qquad (3.23)$$

We can see the diversification can reduce the risk when the correlation coefficient of two financial assets is lower than 1, except this situation $\rho_{12} = \frac{\sigma_1}{\sigma_2}$.

3.5.2 Portfolio of Many Risky Assets

In section 3.5.1, we introduce how the correlation between two assets can affect the risk of portfolio, the lower the correlation the lower the risk. In this section, we extend the analysis to many risky assets rather than two risky assets. The main description of this part is introduced by *Michael G and Jerald E (2011)*.

The portfolio return and variance:

$$E(R_p) = \sum_{i=1}^{N} W_i E(R_i) \tag{3.24}$$

$$\sigma_p^2 = \left(\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i,j=1, i \neq j}^{N} W_i W_j Cov(i,j) \right); \quad \sum_{i=1}^{N} w_i = 1 \tag{3.25}$$

To examine how the portfolio of many risky assets to work and how to cut down the risk of portfolio, we assume there exist equal weight $(1/N)$ in the portfolio for N assets. And $\bar{\sigma}^2$ is the average variance, $\overline{Cov}$ is the average covariance. Under the condition of equal weight $(1/N)$ and average variance/covariance, we can get Equation (3.24) the portfolio's variance.

$$\sigma_p^2 = \left(\sum_{i=1}^{N} W_i^2 \sigma_i^2 + \sum_{i,j=1, i \neq j}^{N} W_i W_j Cov(i,j) \right); \; \sigma_p^2 = \frac{\bar{\sigma}^2}{N} + \frac{(N-1)}{N} \overline{Cov} \tag{3.26}$$

Equation (3.26) states that N become large, the first term on the right side with the denominator N will become lower and lower, which means the contribution of one asset's variance to portfolio variance become insignificant gradually. On the side of second term, the average covariance increase because of N increase. Hence, if the portfolios with a number of stock assets, the covariance among each asset can be regarded as the risk of portfolio.

3.6 Optimal weights

The percentage composition of holding assets in portfolios can be regarded as weights. Hence, optimal weights are the most appropriate percentage holding of each asset. There are

two situations can be considered, one is optimal weights with no riskless assets, another is optimal weights with one riskless asset. In our thesis, we mainly analyze the situation with risky assets.

3.6.1 No Riskless Assets

Based on Markowitz approach, there exist a quadratic optimization problem if we minimize the variance for a given certain expected return. The description mainly introduce by *CHISHOLM, Andrew M (2007)*

$$\min_{w,\lambda,\delta} L(\boldsymbol{w}, \lambda, \delta) = \boldsymbol{w}'\boldsymbol{V}\boldsymbol{w} + \lambda\left(E\left[R_p\right] - \boldsymbol{w}'\bar{\boldsymbol{R}}\right) + \delta(1 - \boldsymbol{w}'\boldsymbol{e}) \tag{3.27}$$

Where,

w is the vector of portfolio weights,

R is the vector of assets return,

$\bar{R}$ is the vector of expected assets return,

e is the vector for all components equal to 1,

V is the variance – covariance matrix of return. We suppose the matrix V is invertible.

The first order conditions are:

$$\frac{\partial L(\boldsymbol{w}, \lambda, \delta)}{\partial \boldsymbol{w}} = 2\boldsymbol{V}\boldsymbol{w} - \lambda\bar{\boldsymbol{R}} - \delta\boldsymbol{e} = 0 \tag{3.28}$$

$$\frac{\partial L(\boldsymbol{w}, \lambda, \delta)}{\partial \lambda} = E\left[R_p\right] - \boldsymbol{w}'\bar{\boldsymbol{R}} = 0 \tag{3.29}$$

$$\frac{\partial L(\boldsymbol{w}, \lambda, \delta)}{\partial \delta} = 1 - \boldsymbol{w}'\boldsymbol{e} = 0 \tag{3.30}$$

3.6.2 One Riskless Asset

If there is a riskless asset, and the return of riskless asset is R_f, using w_0 to represent the percentage of wealth which are invested in the riskless assets. So the budget constraint as follows:

$$\mathbf{w}'\mathbf{e} + w_0 = 1 \leftrightarrow w_0 = 1 - \mathbf{w}'\mathbf{e} \tag{3.31}$$

Hence, the new optimal program:

$$\min \mathbf{w}'\boldsymbol{V}\boldsymbol{w}, with\ w'\bar{R} + (1 - \mathbf{w}'\mathbf{e})\mathrm{R_f} = E\left[R_p\right] \tag{3.32}$$

Associate equation (3.32) with lagrangian function

$$L(w,\lambda) = \mathbf{w}'\mathbf{V}\mathbf{w} + \lambda\left(\mathrm{E}\left[R_p\right] - \boldsymbol{w}'\bar{\boldsymbol{R}}\right) + (1 - \boldsymbol{w}'\boldsymbol{e})R_f) \tag{3.33}$$

Thus, we should solve:

$$\min_{w,\lambda} L(w,\lambda) \tag{3.34}$$

The first order condition is also necessary and sufficient, and shown as follows:

$$\frac{\partial L(\boldsymbol{w},\lambda)}{\partial \lambda} = 2\boldsymbol{V}\boldsymbol{w} - \lambda\left(\bar{\boldsymbol{R}} - \boldsymbol{e}R_f\right) = 0 \tag{3.35}$$

$$\frac{\partial L(\boldsymbol{w},\lambda)}{\partial \lambda} = E\left[R_p\right] - \boldsymbol{w}'\bar{\boldsymbol{R}} - (1 - \boldsymbol{w}'\boldsymbol{e})R_f = 0 \tag{3.36}$$

At the level of $E[R_p]$, the optimal portfolio satisfy:

$$\mathrm{w} = V^{-1}(\bar{\boldsymbol{R}} - \ \mathbf{e}\mathrm{R_f})\frac{E\left[R_p\right] - R_f}{(\bar{\boldsymbol{R}} - \mathbf{e}\mathrm{R_f})^{\boldsymbol{V^{-1}}(\bar{\boldsymbol{R}} - \mathbf{e}\mathrm{R_f})}} \tag{3.37}$$

The variance is given by:

$$\sigma^2\left(R_p\right) = \mathbf{w}'\mathbf{V}\mathbf{w} = \frac{(E\left[R_p\right] - R_f)^{\mathbf{2}}}{J} \tag{3.38}$$

Where $J=B-2AR_f+CR_f^2$ is non – negative,

Hence, its standard deviation is:

$$\sigma\left(R_p\right) = \begin{cases} +\dfrac{\left(E\left[R_p\right] - R_f\right)}{\sqrt{j}} \text{ if } E\left[R_p\right] \geq R_f \\ -\dfrac{(E\left[R_p\right] - R_f}{\sqrt{j}} \text{ if } E\left[R_p\right] \leq R_f \end{cases} \tag{3.39}$$

3.7 Capital Assets Pricing Model

The capital assets price model represents the market-based approach to the estimation of the cost of equity and often used to determine the discount rate for market valuation. It is an equilibrium capital assets pricing model.

If there exist a riskless assets R_f, and the portfolio expected return is composed by market portfolio and risk – free assets, the formula is given as:

$$E(R_p) = w_F \cdot R_F + w_M \cdot E(R_M) \tag{3.40}$$

The choice of proportion for risk – free assets depends on the attitude for risk aversion, as the risk – free asset holds $\sigma_F = 0$, and $\text{cov}(R_F; R_M) = 0$. Hence, the variance holds that:

$$var(R_p) = w_M^2 \cdot var(R_M) \tag{3.41}$$

Based on the assumption: $w_F = (1 - w_M)$, we can get the general capital market line model:

$$E(R_p) = R_F + \frac{E(R_M) - R_F}{\sigma(R_M)} \cdot \sigma(R_p) \tag{3.42}$$

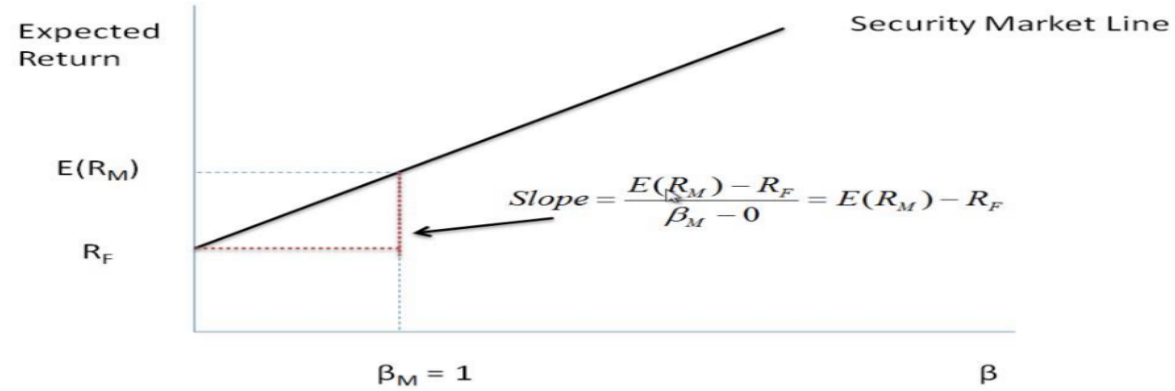
The CAPM model beta version is：

$$E(R_i) = R_F + \beta_i[E(R_M) - R_F] \tag{3.43}$$

Where $E(R_i)$ is the expected return of i - th assets, R_F stands for the risk free rate, a typical risk-free rate of return is a 10-year U.S. government bond, β_i is the stock's sensitivity to the market, it represents the systematic risk which is due to exposition to the market variations, and different company will have various β. $E(R_M)$ is the expected return of the market portfolio. $[E(R_M) - R_F]$ means market risk premium. As we know, the risk free rate and market risk premium are common to all company.

As we can see from Equation (3.43), it is a straight line, so we call it security market line. At equilibrium, all assets are on this line, and in equilibrium, the market portfolio is optimal, which is conducive to passive management based on index funds.

Figure 3.4 Security market line



Source: Jean-Luc, P (2007, p131)

For estimating the risk free rate, in practice we use return of government bonds are the risk free rate, each cash flow should be discounted using a government bond with a similar maturity.

And now we should estimate beta, according to CAPM, a stock's expected return is driven by beta, which measures how much the stocks and market move together. Since beta cannot be directly observed, we have to estimate beta, the most often used method for beta estimation is the regression analysis.

$$\beta^l = \beta^u \left[1 + (1 - t) * \frac{D}{E}\right] \tag{3.44}$$

In general, the cost of equity is higher for a firm than the cost of debt.

3.8 The Standard Performance Measures

Based on the information we introduced before, the security market line and capital market line, we can measure the standard performance such as: Sharpe measure and maximum drawdown. The information is mainly introduce by Jean-Luc, P (2007).

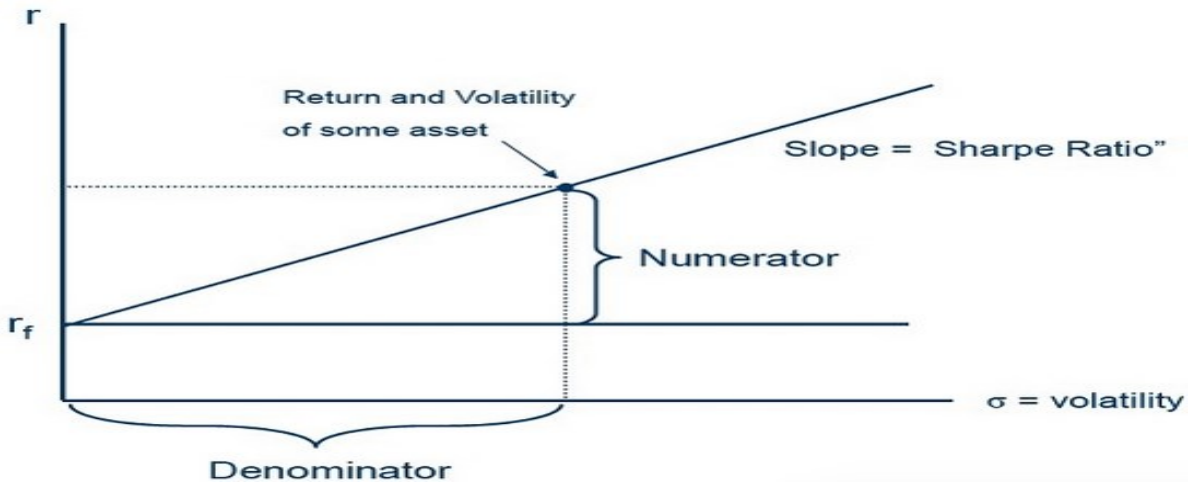
3.8.1 The Sharpe Measure

Sharpe measure is derived from capital market line by William Sharpe, which are applied to measure the performance of financial assets. And can be computed in Equation (3.45).

$$\mathrm{RS} = \frac{\bar{R}_p - R_f}{\sigma(R_p)} \tag{3.45}$$

Sharpe ratio determines portfolio's expected risk premium adjusted by its standard deviation. It is used to compare portfolios. As we can see in Equation (3.45), the numerator $R_p - R_f$ is excess return, which means the return that the portfolio earned that is in excess of the risk free rate of return. All manager of portfolios hope the excess return is positive. The investors look for higher Sharpe ratio to maximize their return for the risk they take.

Figure 3.5 Sharpe ratio



Source: CFA institution

The visual presentation of Sharpe ratio is displayed in figure 3.5. If Sharpe ratio higher than 1, it is good for investor, and represents a better performance for increasing risk. When Sharpe ratio lower than 1, it is not so good.

Moreover, there are points need to attention in the practical application of Sharpe ratio. The effectiveness of the Sharp ratio depends on the assumption that the same risk-free interest rate exist. It only can be valuable in comparison with other different strategies, due to there is no datum point for Sharpe ratio itself.

3.8.2 The Maximum Drawdown

Drawdown is a measurement of decrease from the latest highest point, the measure of drawdown calculated by following Equation (3.46).

$$DD_t = 1 - \frac{W(\tau)}{\max\limits_{t\in(0,\tau)} W(t)} \tag{3.46}$$

Where $W(t)$ is wealth path. Hence, Max drawdown refers to the degree of slippage of account value from the peak point to a subsequent trough. The meaning is that provide information to investors about the worst situation may encounter at any point in time. Here we can see the express of maximum drawdown (MDD) in Equation (3.47).

$$MDD_{0,T} = \max_{t\in(0,\tau)} \left(1 - \frac{W(\tau)}{\max\limits_{t\in(0,\tau)} W(t)} \right) \tag{3.47}$$

Equation (3.47) presents the worst loss of our wealth on historical period. The factors that affect MDD are, the length of time, the frequency of the transaction, the size of the market, the degree of market volatility, disaster or special events. In a complete strategy, maximum drawdown is not a value, but a percentage of the net value of an account, as well as a percentage of the market situation relative to that time.

4. Equity Portfolio Selection using US Dataset

We propose to discover the efficient frontier and optimal composition of stocks in this chapter, by according to the introduction of portfolio management and the information of each alternative model we presented in previous chapter. Then, we apply the weights of each stock and expected return rate to discover the real wealth evolution. In this chapter, we use the real data to optimize our portfolio through the application of *Solver*.

4.1 Data Description

Portfolio investment composed from a number of assets type, such as bonds, stocks, options, foreign currency, financial derivatives, and real estate. While in our thesis, we mainly focus on the analysis of stock portfolios.

For data collection, the sources are available on internet "*Yahoo Finance*"[6]. We should choose stocks which are listed in New York Stock Exchange (NYSE), and concerning Dow Jones Industry Average as the main basis for the selection of stocks as well. As we know, Dow Jones index are the world's oldest stock index and can reflect performance of American market. Actually, the total number of component stocks is 30 in Dow Jones Industry Average, it includes the 30 largest and most well-known U.S. companies. And the dataset of periods we should create is 10 years from January 3rd, 2007 to December 1st, 2016. Hence, we only choose 23 stocks from the 30 component stocks, due to these 23 stocks meet the condition of listing in NYSE and applying the enough historical price data from January 3rd, 2007 to December 1st, 2016. All of these 23 stocks are selected by monthly frequency and adjusted closed price should be used. The units for stock prices are U.S. dollars (USD).

We get 120 monthly adjusted close price in New York stock exchange during January 3rd, 2007 to December 1st, 2016, which is necessary for us to calculate the basic descriptive data such as expected return, standard deviation, variance, and so on. Collecting data is also the first step for us to construct the efficient set and find the optimal composition for portfolios. In following table 4.1, we list the 23 stocks' name that we selected and their abbreviations. And

[6] Source: Stock Price. [Online]. Available on: https://finance.yahoo.com/quote/%5EDJI/components?p=%5EDJI

the collection of 23 stocks' monthly adjusted close prices can be found in Annex 2.

Table 4.1 Abbreviations and name of stocks

Names	**Abbreviation**	**Names**	**Abbreviation**
3M	MMM	McDonald's	MCD
American Express	AXP	Merck	MRK
Boeing	BA	Nike	NKE
Caterpillar	CAT	Pfizer	PFE
Chevron	CVX	Procter & Gamble	PG
Coca-Cola	KO	Travelers	TRV
ExxonMobil	XOM	UnitedHealth	UHN
General Electric	GE	United Technologies	UTX
The Home Depot	HD	Verizon	VZ
IBM	IBM	Wal-Mart	WMT
Johnson & Johnson	JNJ	Walt Disney	DIS
JPMorgan Chase	JPM		

Based on Equation (3.1), we calculate each stock's monthly return. After that, expected (mean) returns of stocks can be computed according to Equation (3.2), it is the average value of stocks' monthly return. The risk of stocks is expressed by variance and standard deviation, we can compute them by Equation (3.3) and Equation (3.4) respectively. Table 4.2 displays the monthly expected (mean) returns and risks during our analyzed period from January 3rd, 2007 to December 1st, 2016.

Asset	*E(Ri)*	σ_i	Assets	*E(Ri)*	σ_i	Assets	*E(Ri)*	σ_i	Assets	*E(Ri)*	σ_i
3M	0.96%	5.44%	XOM	0.38%	4.67%	MCD	1.11%	4.13%	UHN	1.03%	8.15%
AXP	0.34%	10.24%	GE	0.19%	8.59%	MRK	0.56%	6.39%	UTX	0.60%	5.78%
BA	0.67%	7.81%	HD	1.22%	6.42%	NKE	1.31%	6.53%	VZ	0.74%	5.21%
CAT	0.54%	10.16%	IBM	0.61%	5.47%	PFE	0.53%	5.47%	WMT	0.51%	4.60%
CVX	0.70%	5.91%	JNJ	0.72%	4.12%	PG	0.46%	4.36%	DIS	1.05%	6.56%
KO	0.71%	4.62%	JPM	0.65%	8.91%	TRV	0.95%	5.34%			

Table 4.2 Monthly expected (mean) returns and risks

Seen from table 4.2, NKE achieved the highest expected (mean) return which is 1.31%; the lowest expected (mean) return is 0.19%, which can be found in stock GE. The greatest risk is appeared in stock AXP with standard deviation value of 10.24%; while the lowest standard deviation 4.12% comes from JNJ. In general, higher risk corresponding with higher return. However, we can discover that the highest risk stock AXP with relative lower return; while the

risk of NKE is not very high compared with other stocks.

Another necessarily input data is covariance matrix. Equation (3.5) in chapter 3 provides the information about how to calculate covariance. Table 4.3 presented that all the results of covariance matrix are positive, which stands for any two variables of the 23 component stocks change with the same direction. The greater the covariance value, the greater the degree of changing in the same direction of any two stocks and vice versa. However, if there is negative value of covariance, it means the trends of two stocks are reversed, if the return of one stock becomes higher, the another stock's return will tend to be lower. The greater the absolute value of the covariance, the closer the relationship between the two assets returns; the less absolute value suggests that the relationship between the two assets returns are alienated.

Table 4.3 Covariance matrix

	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS
MMM	0.003	0.004	0.002	0.004	0.002	0.001	…	0.002	0.002	0.002	0.001	0.000	0.002
AXP	0.004	0.010	0.005	0.006	0.002	0.001	…	0.002	0.003	0.004	0.001	0.001	0.004
BA	0.002	0.005	0.006	0.003	0.002	0.001	…	0.002	0.003	0.003	0.001	0.000	0.003
CAT	0.004	0.006	0.003	0.010	0.003	0.002	…	0.002	0.002	0.004	0.002	0.001	0.004
CVX	0.002	0.002	0.002	0.003	0.003	0.001	…	0.001	0.001	0.002	0.001	0.000	0.002
KO	0.001	0.001	0.001	0.002	0.001	0.002	…	0.001	0.000	0.001	0.001	0.001	0.001
…	…	…	…	…	…	…	…	…	…	…	…	…	…
TRV	0.002	0.002	0.002	0.002	0.001	0.001	…	0.003	0.001	0.002	0.001	0.001	0.002
UHN	0.002	0.003	0.003	0.002	0.001	0.000	…	0.001	0.007	0.003	0.001	0.000	0.002
UTX	0.002	0.004	0.003	0.004	0.002	0.001	…	0.002	0.003	0.003	0.001	0.000	0.002
VZ	0.001	0.001	0.001	0.002	0.001	0.001	…	0.001	0.001	0.001	0.003	0.001	0.001
WMT	0.000	0.001	0.000	0.001	0.000	0.001	…	0.001	0.000	0.000	0.001	0.002	0.000
DIS	0.002	0.004	0.003	0.004	0.002	0.001	…	0.002	0.002	0.002	0.001	0.000	0.004

As illustrated by table 4.3, the covariance matrix illustrates that the highest covariance is appeared between AXP and AXP, the covariance of this stock is equal to 0.01048. And covariance between BA and BA is 0.006, the covariance value is same with the variance of BA. Hence, the same stock's covariance is equal to its variance. In generally, the covariance between any two other assets is lower than the same stock's covariance, it indicates the risk of investing in two different stock assets is less than only investing in one same stock. Such as the covariance between BA and CAT, it is 0.003, we can find there is a decrease of 0.3% compared with

covariance between BA and BA. Hence, the diversification of assets can effectively eliminate the investment risk and keep our principal safer.

4.2 Construction of Optimal Portfolio

To construct the optimal portfolio, it is necessary for us to comply with the basic procedures. For investors, they firstly need to select a stock exchange that they want to invest in, in our thesis, New Stock Exchange (NYSE) is chosen. Moreover, in order to ensure the reliability of our analysis, we select 23 component stocks of Dow Jones Industry Average (DJIA) as the investment assets. Although DJIA has a total of 30 component stocks, only 26 listed on NYSE, we only can apply 23 stocks' completely historical adjust close price in our investment horizon from January 3rd, 2007 to December 1st, 2016. The reason for selecting the components of DJIA is that all of these stocks in DJIA are high capitalization, ignoring the inflation and avoiding the problem of high selectivity. For new investors, they usually cannot identify which stock is better from a lot of stocks, hence, DJIA is a helpful index for using. Furthermore, all needed input data we have presented in subchapter 4.1. Now, we can construct the portfolio.

4.2.1 Markowitz Model

Markowitz model also refers to mean-variance model. In order to apply Markowitz model to find the optimal portfolio, we assume there is no riskless assets and no permission of short selling. Furthermore, we search the minimal standard deviation and maximum expected return of portfolios under given constraints, the objective function and constraints we have introduced in Equation (3.11). Due to there are 119 months in our investment horizon, we get 118 intervals.

The change of monthly weights is displayed by table 4.4. Under the constraints of sum of assets weights should be equal to 1, we can find we have not invested into all 23 stocks, the weights are 0 for those assets express that they are not worth to invest. To better analysis, we divided our investment horizon into two parts, we consider the first five years as our first part, which begin from February 2nd, 2007 until December 1st, 2011; the second five years part as late investment period. It not hard to find the weights of WMT and MCD are much higher than other stocks, it occupied almost 50% during our early investment period. The highly occupied

weights of MCD is caused by the higher profitability and efficient assets management of this company. The average assets turnover tends to increase from 2007 to 2014, it means the usage of MCD's assets is really efficiency and indicates the more invested in assets will generate higher revenue.

Table 4.4 The composition of each asset in Markowitz model

	2/1/07	3/1/07	4/2/07	…	1/4/10	2/1/10	…	7/1/14	8/1/14	…	10/3/16	11/1/16	12/1/16
MMM	0.00%	0.00%	0.00%	…	2.04%	2.21%	…	0.74%	0.00%	…	0.00%	0.00%	0.00%
AXP	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
BA	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
CAT	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
CVX	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
KO	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
XOM	13.80%	13.56%	13.31%	…	4.15%	3.84%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
GE	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
HD	4.78%	4.96%	5.10%	…	10.01%	10.13%	…	24.39%	24.84%	…	12.72%	6.36%	0.00%
IBM	12.85%	12.79%	12.73%	…	10.54%	10.47%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
JNJ	0.77%	0.82%	0.90%	…	1.94%	1.94%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
JPM	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
MCD	21.24%	21.67%	22.12%	…	36.76%	37.21%	…	60.04%	60.05%	…	0.00%	0.00%	0.00%
MRK	1.15%	1.08%	0.96%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
NKE	1.74%	1.81%	1.88%	…	4.02%	4.06%	…	11.37%	12.00%	…	87.28%	93.64%	100%
PFE	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
PG	14.00%	13.70%	13.42%	…	3.67%	3.36%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
TRV	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
UHN	0.43%	0.48%	0.55%	…	1.90%	1.92%	…	3.08%	3.11%	…	0.00%	0.00%	0.00%
UTX	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
VZ	5.42%	5.42%	5.44%	…	5.40%	5.38%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
WMT	23.82%	23.71%	23.58%	…	19.57%	19.46%	…	0.39%	0.00%	…	0.00%	0.00%	0.00%
DIS	0.00%	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	…	0.00%	0.00%	0.00%
Sum Xi	1	1	1	…	1	1	…	1	1	…	1	1	1

According to the profitability of MCD, return on assets increase from 2007 to 2013 but decrease from 2014 to 2016. In other words, the company MCD performs well from 2007 to 2014, that's why we invest more money into MCD during our early investment period. However, after 2014, the financial situation of MCD getting worse than before, due to more and more people are aware of the importance of health. For the worse financial situation, we reduce the investing weight gradually and stop to invest in MCD from July 1st, 2016. WMT vigorously develop the Chinese market, in September 2011 WMT acquired Trust – Mart. The net sale of

WMT growth year by year, while the increase percentage of net sale decreased 10% from 2007 to 2011. We also can find the debt of WMT tends to rise year by year from checking the annual report, which means there can be a risk of liquidity. Obviously, WMT is getting more and more risky but the expected return has not significantly increase. This is why the proportion of investing in WMT less and less.

During our late investment period, the weights of investing in HD and NKE is growing. Especially for stock NKE, there is more than 50% proportion of our investment from the beginning of 2016. As we know, earnings per share (EPS) is the most important financial measure of the company's profitability, it can reflect the performance of common stock. By comparing the EPS of NKE, EPS for fiscal year 2016 increased 83% to 2.16 USD from 1.18 USD for year 2012, it illustrates that shareholder can enjoy a higher net profit for holding this stock. With the development of the Internet, NKE has taken online and offline sales strategy at the same time, which is one reason of higher return as well.

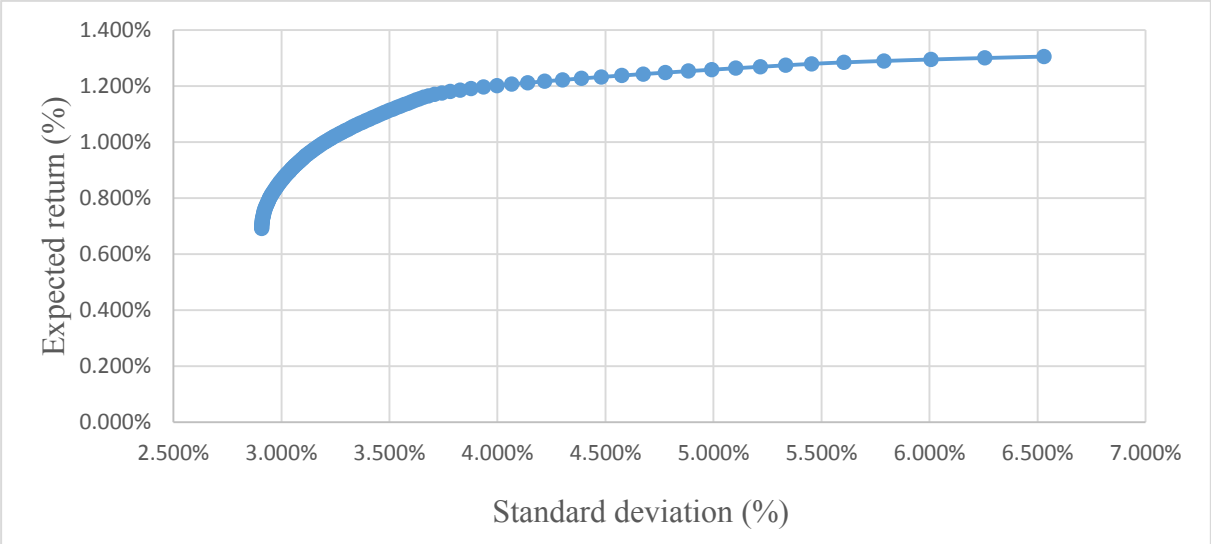


Figure 4.1 Efficient set of Markowitz model

As displayed by figure 4.1, the efficient frontier is expressed by the blue line. It illustrates the composition of efficient portfolio on the basis of investors' different expectations about return and risk. Based on the information of input data (expected mean return and standard deviation of each asset according to last 10 year's stock adjust close price), we can see the efficient frontier of portfolio expected return is in the range between 0.6% and 1.4%; the portfolio standard deviation on efficient frontier with lower limit 2.5% and higher limit 7%. Depending on different level of risk averse, investors can select the optimal portfolio to

maximum their own profit. The results is apparent in figure 4.1. Investors consider that investing all amount of capital into NKE is the optimal choice, if they are with the minimal level of risk aversion. Thus, stock NKE with the highest expected return of 1.306% and largest standard deviation (risk) of 6.530%. On the other side, investors should invest into XOM, HD, IBM, JNJ, MCD, MRK, NKE, PG, UHN, VZ, WMT with weights 13.8%, 4.78%, 12.85%, 0.77%, 21.24%, 1.15%, 1.74%, 14%, 0.43%, 5.42%, 23.82% respectively, if investors with the maximal risk aversion and prefer to minimize the risk. We can find that there is a lot of stocks with the minimum risk, not only one stock, it is caused by the covariance is less than 1.

4.2.2 Black's Model

Black's model is one type of mean-variance model as well. It is similar with Markowitz model, but in this case, the assumption of short sale of assets is feasible. However, it is still prohibited to invest into risk-free assets.

The weights of stocks can be positive or negative in Black's model with the permission of short selling, all results is described in table 4.5. Although the negative weights can be exist, it could not lower than -1. By taking the opportunity to short selling can be observed by analyzing the results of each efficient portfolio. The results illustrate by the negative composition for stock AXP, CAT, CVX, GE, JNJ, PFE, those with the lower expected mean return 0.340%, 0.544%, 0.698%, 0.194%, 0.717%, 0.532% respectively. Subsequently, after run out of the potential of shorting selling of these stocks, stock PG, UTX, VZ, WMT, those with lower expected return as well, will be contained in the portfolios. Table 4.5 indicates that we use the cash earned by shorting selling of lower expected assets to invest into stocks with higher expected return.

During the early investment period, we mainly invested into three assets MMM, MCD and DIS. The free cash flow of company 3M increase by 37% from 2,853 million USD in 2007 to 3905 USD for year 2011. Free cash flow is a financial method used to measure the amount of cash that an enterprise actually holds, which means the maximum amount of money available to shareholders (and creditors) without prejudice to the survival and development of the company. With the free cash flow of MMM increases, shareholders can get a higher return, furthermore we can get the more return on investment capital. The most important is MMM

paid 4 billion USD to shareholders in 2011 through the methods of stock repurchases and dividends. Hence, the weight of investing in MMM is so high in our early investment period. In 2009, Disney (DIS) acquired Marvel Entertainment; Disney acquired Lucas Ltd in 2011, at the time of company delivery, Disney issued about 40 million shares. It is good choice for investing in DIS.

Table 4.5 The composition of each assets in Black's model

	2/1/07	3/1/07	4/2/07	…	11/1/11	12/1/11	2/1/12	…	9/1/16	10/3/16	11/1/16	12/1/16
MMM	11.88%	15.87%	19.87%	…	238.04%	241.29%	248.38%	…	-100.00%	-100.00%	-100.00%	-100.00%
AXP	-0.93%	-2.93%	-4.93%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
BA	0.43%	0.07%	-0.26%	…	-49.62%	-52.52%	-58.23%	…	-100.00%	-100.00%	-100.00%	-100.00%
CAT	-8.58%	-8.33%	-8.07%	…	-15.69%	-17.39%	-20.97%	…	-100.00%	-100.00%	-100.00%	-100.00%
CVX	-5.48%	-4.29%	-3.09%	…	-61.55%	-63.96%	-68.85%	…	-100.00%	-100.00%	-100.00%	-100.00%
KO	1.38%	1.33%	1.29%	…	-83.20%	-88.13%	-97.67%	…	-100.00%	-100.00%	-100.00%	-100.00%
XOM	24.10%	18.44%	12.80%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
GE	-17.00%	-18.91%	-20.84%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
HD	11.49%	13.66%	15.83%	…	177.50%	181.50%	189.38%	…	334.37%	189.58%	44.79%	-100.00%
IBM	16.27%	15.45%	14.63%	…	-37.49%	-39.60%	-43.96%	…	-100.00%	-100.00%	-100.00%	-100.00%
JNJ	-2.35%	-1.05%	0.25%	…	-34.45%	-40.02%	-50.80%	…	-100.00%	-100.00%	-100.00%	-100.00%
JPM	1.42%	1.82%	2.23%	…	-28.23%	-31.07%	-36.80%	…	-100.00%	-100.00%	-100.00%	-100.00%
MCD	17.61%	23.07%	28.52%	…	405.22%	415.81%	436.62%	…	-100.00%	-100.00%	-100.00%	-100.00%
MRK	5.11%	4.40%	3.67%	…	-78.20%	-80.73%	-86.20%	…	-100.00%	-100.00%	-100.00%	-100.00%
NKE	11.76%	12.27%	12.78%	…	92.01%	94.48%	99.29%	…	1865.63%	2010.42%	2155.21%	2300.00%
PFE	-3.69%	-7.27%	-10.84%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
PG	19.57%	16.70%	13.81%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
TRV	-11.05%	-9.64%	-8.25%	…	64.07%	65.11%	66.71%	…	-100.00%	-100.00%	-100.00%	-100.00%
UHN	1.61%	3.12%	4.63%	…	87.11%	88.79%	92.13%	…	-100.00%	-100.00%	-100.00%	-100.00%
UTX	4.60%	3.09%	1.54%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
VZ	8.08%	7.60%	7.11%	…	-51.79%	-53.98%	-58.22%	…	-100.00%	-100.00%	-100.00%	-100.00%
WMT	19.72%	18.57%	17.42%	…	-100.00%	-100.00%	-100.00%	…	-100.00%	-100.00%	-100.00%	-100.00%
DIS	-5.96%	-3.03%	-0.11%	…	176.27%	180.44%	189.19%	…	-100.00%	-100.00%	-100.00%	-100.00%
Sum Xi	100.00%	100.00%	100.00%	…	100.00%	100.00%	100.00%	…	100.00%	100.00%	100.00%	100.00%

As presented in table 4.5, we propose to invest more money into stock HD during our late investment period. The operating income of HD rises by 102% from 6,661 million USD for fiscal year 2012 to 13,427 USD in 2013. Home Depot is aware that China is a big and growing market with more than 1.3 billion people, there is a huge potential in China. Therefore, they vigorously develop the Chinese market. As a result of exploiting overseas markets, the sale of HD growth year by year, hence, there is a good performance expectation in HD. This is the reason for why we should invest more in stock HD.

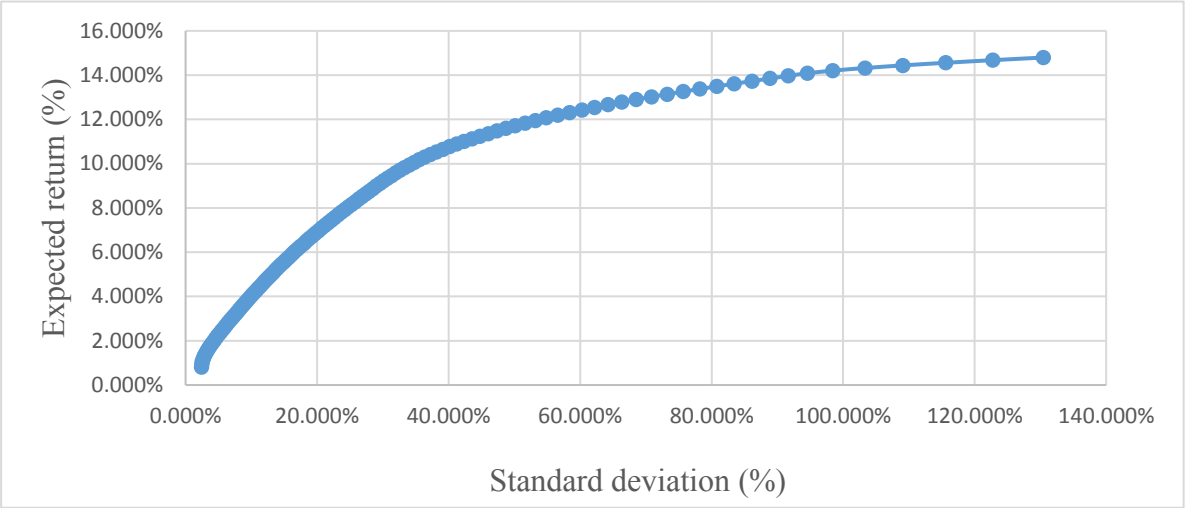


Figure 4.2 Efficient set of Black's model

The efficient frontier of Black's model is depicted in figure 4.2. The first boundary with 2.439% risk and 0.816% expected return for the minimal risk portfolio, which will be selected by investors with high level of risk aversion; the second boundary with 14.796% expected return and 130.475% risk, which is chosen by minimum risk aversion investors. In this case of maximum standard deviation portfolio, investors prefer to pursue higher risk and higher return, hence, we only invest into one equity (NKE) with the highest expected return. Due to the short selling is allowed for us, we not only invest all initial money that we have into NKE, but also the total amount of money we gained from selling the other risky assets short.

4.2.3 The Mean-VaR Model

As the description of Markowitz model in section 4.2.1, we have seen the risk measure of Markowitz model is standard deviation. However, in this subchapter, we will discuss another risk measure method – Value at Risk. And compare the portfolio optimization under different significance level.

As we know, VaR estimates the amount of investment with the specified confidence level of 1-α that investors may lose. Higher the VaR represents higher the risk of portfolio under the condition of same confidence level. Hence, we want to minimize the VaR in general. Due to we try to maintain this thesis well-arranged, the basic assumption in mean-VaR is same with Markowitz model (there is no short selling and without risk free asset), the parameter (return of stocks, number of equities, sum of weights) is constant as well. Compared to traditional risk

measures (represented by standard deviation), VaR focuses on considering downside risks of important assets, especially the scale of losses in extreme case.

We assume the return is normally distributed. Then the optimal composition can be determined by taking advantage of minimizing VaR. In our thesis, we assume the weight of each share should not be less than 1%, suppose two particular significance level 1% and 15%.

Table 4.6 Correlation matrix

	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS
MMM	1	0.654	0.541	0.637	0.564	0.403	…	0.534	0.358	0.654	0.377	0.167	0.615
AXP	0.654	1	0.595713	0.600	0.365	0.303	…	0.386	0.376	0.616	0.198	0.128	0.620
BA	0.541	0.596	1	0.407	0.396	0.338	…	0.461	0.508	0.681	0.268	0.107	0.649
CAT	0.637	0.600	0.407	1	0.569	0.382	…	0.439	0.288	0.656	0.365	0.206	0.584
CVX	0.564	0.365	0.396	0.569	1	0.403	…	0.469	0.212	0.498	0.430	0.083	0.561
KO	0.403	0.303	0.338	0.382	0.403	1	…	0.444544	0.13291	0.295234	0.491167	0.347336	0.418871
…	…	…	…	…	…	…	…	…	…	…	…	…	…
TRV	0.534	0.386	0.461	0.439	0.469	0.445	…	1	0.324	0.526	0.402	0.374	0.512
UHN	0.358	0.376	0.508	0.288	0.212	0.133	…	0.324	1	0.544	0.213	0.045	0.453
UTX	0.654	0.616	0.681	0.656	0.498	0.295	…	0.526	0.544161	1	0.251	0.162	0.659
VZ	0.377	0.198	0.268	0.365	0.430	0.491	…	0.402	0.212659	0.251	1	0.268	0.332
WMT	0.167	0.128	0.107	0.206	0.083	0.347	…	0.374	0.044634	0.162	0.268	1	0.153
DIS	0.615	0.620	0.649	0.584	0.561	0.419	…	0.512	0.453266	0.659	0.332	0.153	1

The correlation among each stock is presented in table 4.6. According to correlation matrix, we can obtain new covariance matrix. We use VaR as our new method to estimate risk, therefore, there will be exist a new boundary set of portfolio compared with Markowitz model.

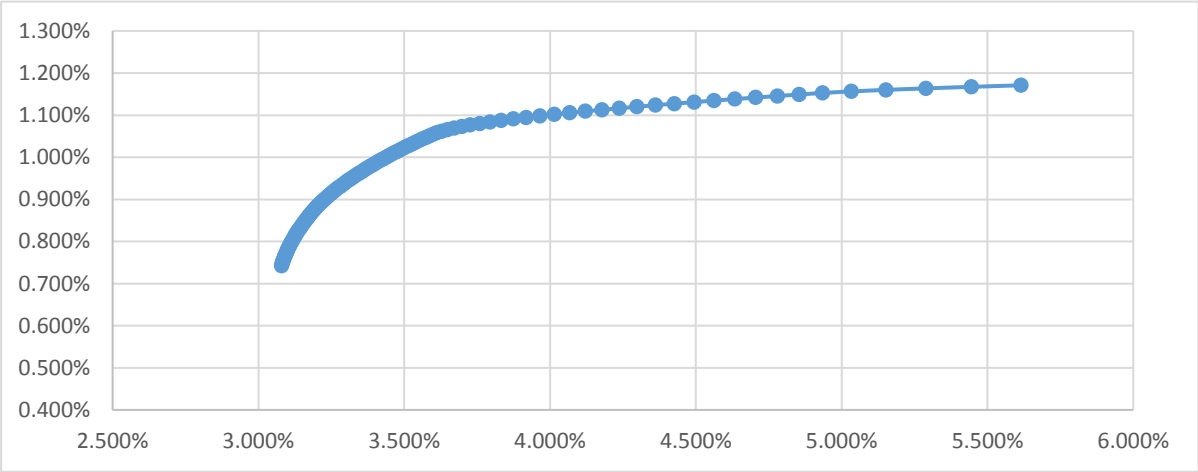


Figure 4.3 Efficient set of mean-VaR model ($\alpha = 0.01$)

Depicted by figure 4.3, the minimal VaR portfolio with expected return of 0.742 % should be selected by investors who with the highest level of risk aversion. For effective analysis, we combine the optimal composition of each stock in portfolio that shown in table 4.7. Therefore, we can find the relative share of each assets respectively are 7.09% of XOM, 4.23% of HD, 9.95% of IBM, 26.50% of MCD, 7.99% of PG, 5.09% of VZ and 23.15% of WMT, the other stocks keep the same relative weights of 1%. By contrast, investors should invest into the maximal expected return portfolio with 5.615% risk to obtain the highest expected return of 1.171%. Being same with Markowitz model, investors whose version of risk is minimum, should invest all amount of resources into NKE with the maximal weight of 100%, due to the expected mean return of NKE is the highest.

	2/1/07	3/1/07	4/2/07	5/1/07		11/1/11	12/1/11	1/3/12	2/1/12		9/1/16	10/3/16	11/1/16	12/1/16
MMM	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
AXP	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
BA	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
CAT	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
CVX	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
KO	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
XOM	7.09%	6.86%	6.68%	6.45%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
GE	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
HD	4.23%	4.35%	4.55%	4.69%	…	13.65%	13.86%	14.07%	14.28%	…	14.32%	9.88%	5.44%	1.00%
IBM	9.95%	9.96%	9.86%	9.86%	…	4.28%	4.09%	3.90%	3.72%	…	1.00%	1.00%	1.00%	1.00%
JNJ	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
JPM	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
MCD	26.50%	26.91%	27.24%	27.61%	…	44.90%	45.20%	45.51%	45.81%	…	1.00%	1.00%	1.00%	1.00%
MRK	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
NKE	1.00%	1.00%	1.00%	1.00%	…	4.84%	4.94%	5.04%	5.14%	…	64.68%	69.12%	73.56%	78.00%
PFE	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
PG	7.99%	7.77%	7.60%	7.46%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
TRV	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
UHN	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
UTX	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
VZ	5.09%	5.07%	5.08%	5.09%	…	2.26%	2.14%	2.01%	1.89%	…	1.00%	1.00%	1.00%	1.00%
WMT	23.15%	23.08%	23.00%	22.85%	…	13.06%	12.77%	12.47%	12.17%	…	1.00%	1.00%	1.00%	1.00%
DIS	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%	…	1.00%	1.00%	1.00%	1.00%
Sum Xi	1	1	1	1	…	1	1	1	1	…	1	1	1	1

Table 4.7 Composition of each assets in mean-VaR model ($\alpha = 0.01$)

The composition of efficient portfolios are presented in table 4.7, the fully results is displayed by Annex 7. In this case, 1% probability level is supposed. It expressed the weights

of investing into stock WMT, MCD, IBM is higher during the early investment period, record more than 65% of initial resources. Here, we focus on the analysis of IBM. During the past 5 years, earnings per share of stock IBM have increased to 13.44 USD and risen approximately by threefold; free cash flow has increased by 166 billion USD. There mainly are three segments for IBM: software, hardware, and services, where the software revenue accounted for half of IBM's overall revenue. Most importantly, they return revenues to shareholders with 107 billion USD in 2010. We tend to invest more resources into NKE, MCD and HD during the late investment period.

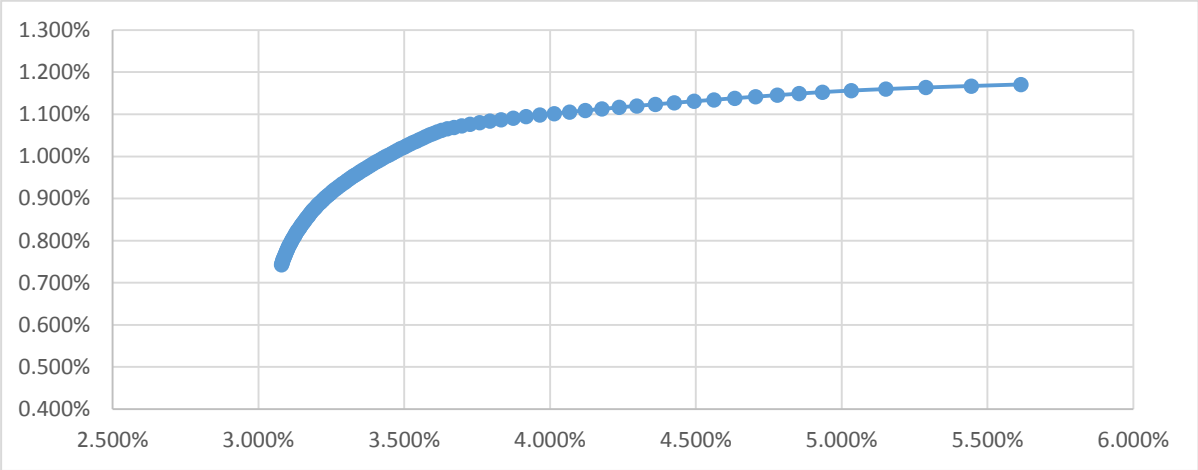


Figure 4.4 Efficient set of mean-VaR model ($\alpha = 0.01$)

Depicted by figure 4.4, the minimal VaR portfolio with expected return of 0.742 % should be selected by investors who with the highest level of risk aversion. Therefore, the relative share of each assets respectively are 7.09% of XOM, 4.23% of HD, 9.95% of IBM, 26.50% of MCD, 7.99% of PG, 5.09% of VZ and 23.15% of WMT, the other stocks keep the same relative weights of 1%. By contrast, investors should invest into the maximal expected return portfolio with 5.615% risk to obtain the highest expected return of 1.171%. Being same with Markowitz model, investors whose version of risk is minimum, should invest all amount of resources into NKE with the maximal weight of 100%.

Next, we set different significant level of 15%. The results partially displayed by table 4.8.

The optimal composition of portfolio is different under significance level of 15% compared with probability level of 1%. Investors with maximal risk aversion should choose the efficient portfolio with minimal VaR, compared with 1% significance level, the share of relative

increase in each stocks are 8.16% of MCD, 3.31% of HD, 0.76% of NKE and 0.12% of VZ; while there are some decrease of relative share, they are respectively 4.72% of XOM, 0.84% of IBM, 4.35% of PG and 2.43% of WMT. However, for investors whose version to risk is minimal, they will select to invest into NKE with high proportion of 78%, in both significance level.

Table 4.8 The composition of each assets in mean-VaR model ($\alpha = 0.15$)

	2/1/07	3/1/07	4/2/07	5/1/07	..	11/1/11	12/1/11	1/3/12	2/1/12	..	9/1/16	10/3/16	11/1/16	12/1/16
MMM	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
AXP	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
BA	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
CAT	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
CVX	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
KO	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
XOM	2.37%	2.21%	2.11%	2.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
GE	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
HD	7.54%	7.61%	7.72%	7.80%	..	16.20%	16.37%	16.53%	16.70%	..	11.52%	7.97%	4.43%	1.00%
IBM	9.11%	9.08%	9.04%	8.97%	..	1.90%	1.73%	1.56%	1.38%	..	1.00%	1.00%	1.00%	1.00%
JNJ	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
JPM	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
MCD	34.66%	34.93%	35.23%	35.45%	..	48.48%	48.67%	48.88%	49.07%	..	1.00%	1.00%	1.00%	1.00%
MRK	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
NKE	1.76%	1.83%	1.85%	1.95%	..	6.12%	6.21%	6.31%	6.41%	..	67.48%	71.03%	74.57%	78.00%
PFE	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
PG	3.64%	3.46%	3.25%	3.10%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
TRV	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
UHN	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
UTX	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
VZ	5.20%	5.21%	5.21%	5.20%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%
WMT	20.72%	20.66%	20.59%	20.51%	..	9.29%	9.01%	8.72%	8.45%	..	1.00%	1.00%	1.00%	1.00%
DIS	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%	..	1.00%	1.00%	1.00%	1.00%

The results of relatve change between two different significant level is apparent in figure 4.5. If we suppose different probability level, the fluctuation of MCD is the highest with postive change; while the largest negative fluctuation comes from XOM, under the selection of minimal risk portfolio. The results indicates, with the change of confidence level, we prefer to invest more pripical into stock MCD with the lower confidence level; and invest less to XOM with the higher significance level.

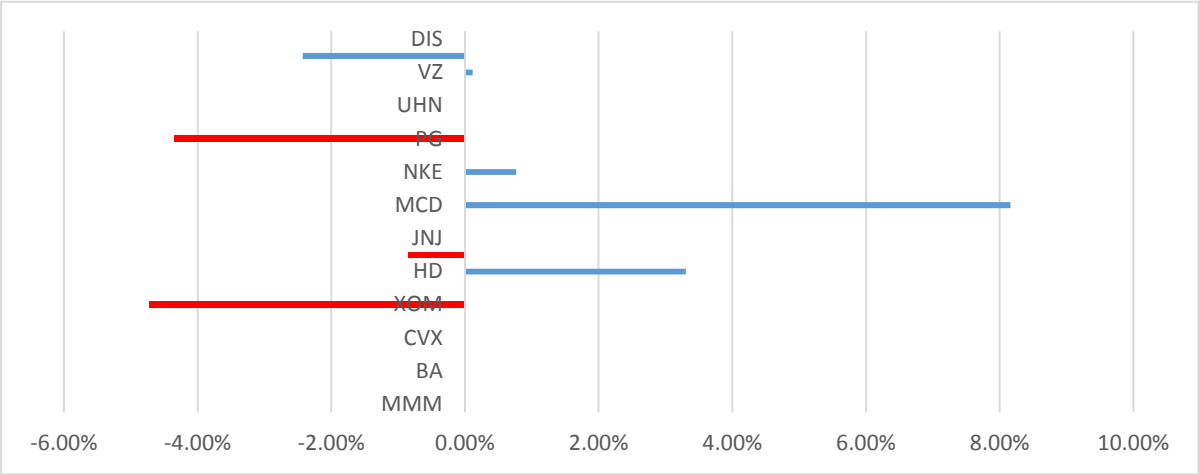


Figure 4.5 The evolution of relative share in each assets

. Figure 4.6 displays the construction of efficient set, based on investors' different expectations about return and risk. We can see the first boundary with 3.14% risk and 0.829% expected return; the second boundary on efficient frontier with a standard deviation of 5.615% and 1.171% of return. The range of expected return is 0.342%, corresponding with the 3.475% range of risk.

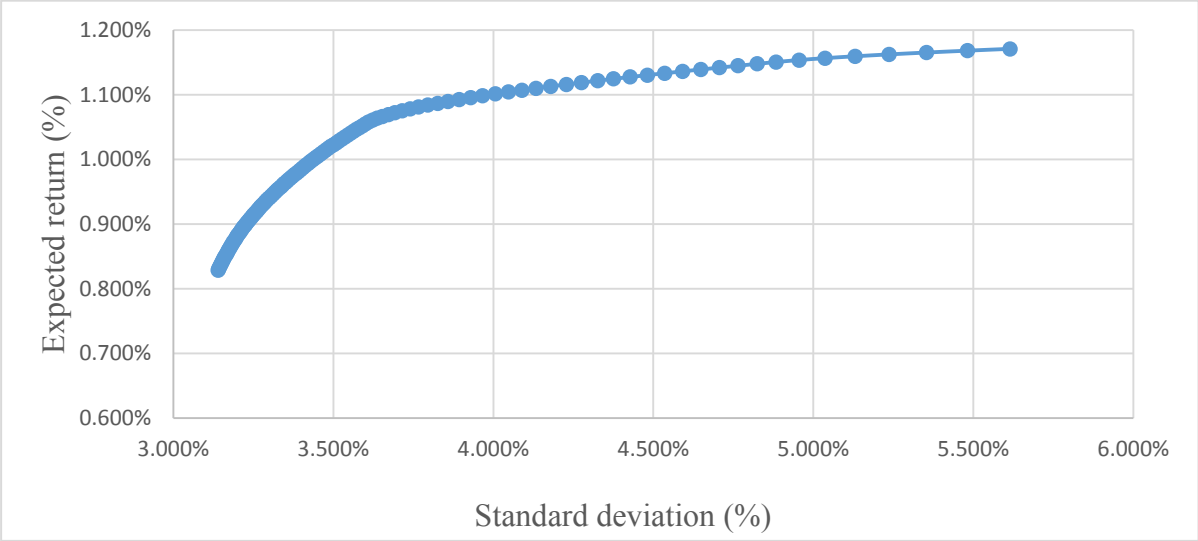


Figure 4.6 The construction of efficient set with mean-VaR model ($\alpha = 0.15$)

4.3 Interpretation of Each Model

In previous subchapter 4.2, we described three models optimal composition and efficient set. Here, we propose to analyze evolution of wealth by virtue of the different combination of relative share on each stock. It is necessary for us to compare real wealth evolution and expected evolution for the purpose of testing the composition efficiency. Moreover, under the

consideration of describing the evolution on wealth more vividly, we suppose the total amount of initial resources we can invest is 1,000 USD, on February 3rd, 2007.

Model 1: Markowitz model

As illustrated by table 4.1, the composition pf each asset. Hence, we can calculate the real return rate based on the stock monthly return and relative weight, it can be deduced by Equation (3.13). Obtained real return rate is useful to compute our real wealth. The visual representation of expected and real wealth evolution with selection of Markowitz model is depicted in figure 4.7.

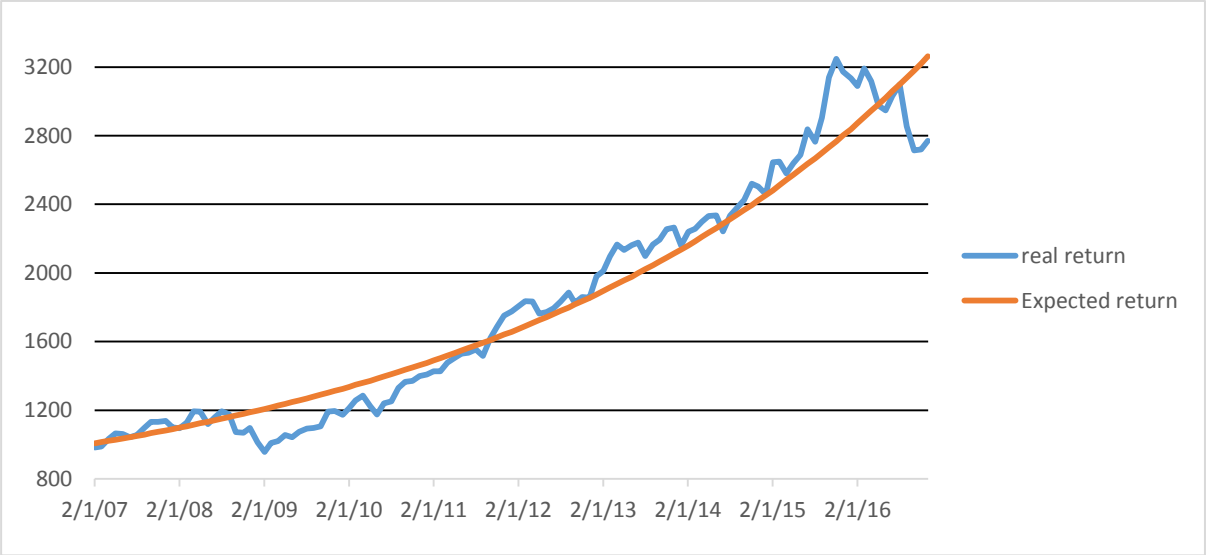


Figure 4.7 Wealth evolution with Markowitz model

Seen from figure 4.7, the whole tendency is upward in both expected and real items. For more intuitive analysis, we divide the wealth path into three parts, the first part is from February 2007 to February 2009; the second phase lasting form March 2009 to December 2015; fiscal year 2016 constitutes the third part.

In the first phase, the real wealth is relatively lower, the lowest point appeared in February 2009 with real return of 957.09 USD, which illustrates we suffer the risk of losing principal in this period, due to our initial investment is 1,000 USD. This situation is caused by the negative influence of financial crisis. In financial year 2008 and 2009, the global financial crisis broke out, U.S. real gross domestic product (GDP) decrease by 6.3% in the fourth quarter of 2008, subsequently, down by 5.7% in the first quarter of 2009 as well. Which result in the Dow Jones index drop sharply, on October 6th, 2008, this index below 10,000 points, to October 9th, the

index fell below 9000 points. Actually, due to the impact of the U.S. subprime mortgage, index began to fall in November 2007.

Rising trend appeared in the second phase, it is a good opportunity for us to do investment during this period. Due to the positive effect of quantitative easing and re-industrialization policy, economy of United States gradually recover from 2010. Therefore, since October 2011, our real wealth has exceeded our expected return. Furthermore, the highest real wealth of 3248.48 USD occurred in this phase on November 2nd, 2015, which results from the rise of employment rate, US unemployment rate fell to 5.3%, close to the full employment level.

The third phase tends to decline, from the real wealth of 3174 USD to 2771 USD, decreasing by 14.5% in financial year 2016. The main reasons for negative growth are the following aspects: the sharply decrease of international oil price led energy production decrease, what limits the rise of US GDP; another reason for GDP decrease is results from less competitiveness of export, due to the US dollar continued to appreciation. For these negative effect, our final wealth (2771 USD) lower than expectation (3263 USD).

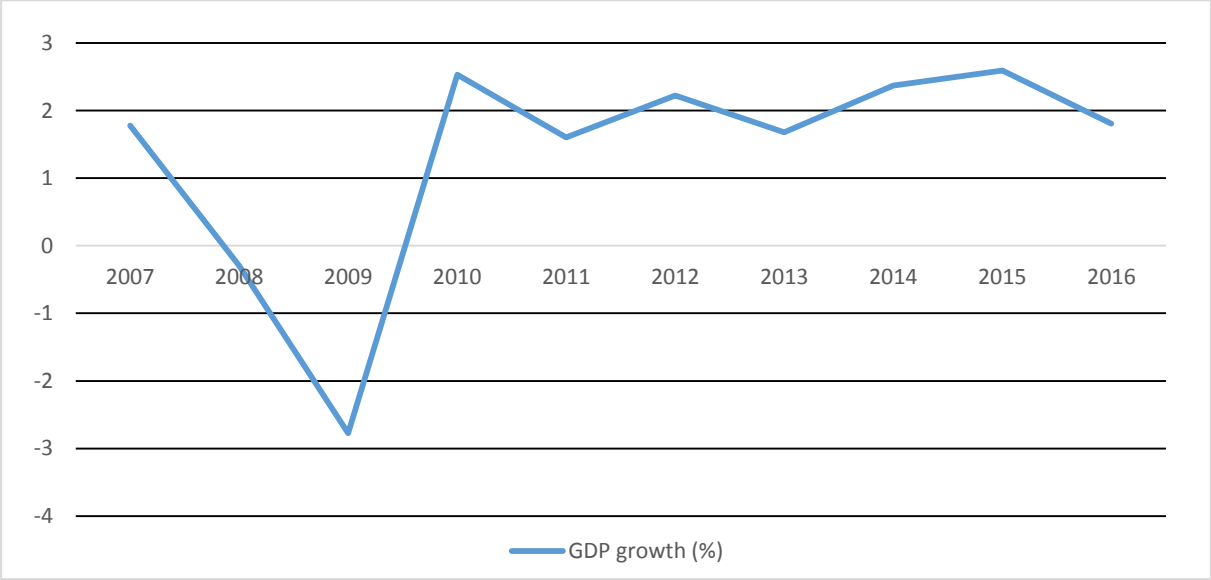


Figure 4.8 United States GDP growth

Model 2: Black's model

The visual representation of wealth path with Black's model is presented by figure 4.9. As can be seen, the evolution of real wealth path is much more fluctuate than expected wealth. The highest real wealth is appeared in November 2015 with 3390.174 USD; by contrast, the lowest real wealth is 907.639 USD. The difference between this two points is 2482.535 USD, which

means, there is a 273.5% increase in our wealth during our investment period. Compared with expected wealth, whose fluctuation between two extreme points is 225.23%, real wealth with almost 50% distance. For better analysis, we propose to divide the evolution path of real wealth into two phase, the first part is before financial 2013, the second phase from 2013 to 2016.

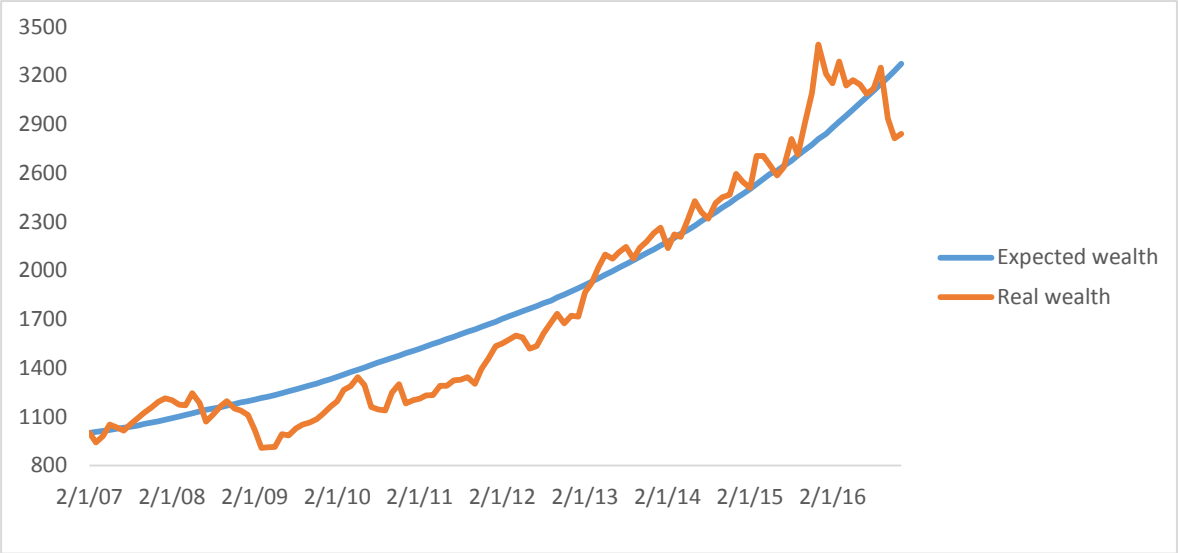
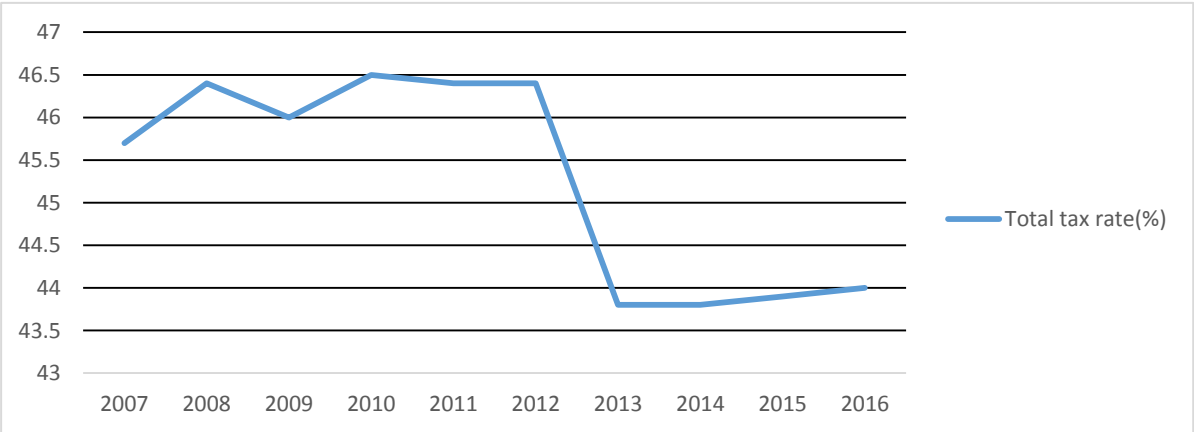


Figure 4.9 Wealth evolution with Black's model

The first stage mostly consists of real wealth that is less than expected wealth. For better analysis, we combine the fluctuation of tax rate in the United States from 2007 to 2016, which is depicted in figure 4.10. During this period, US government has a tight fiscal policy with higher tax rate and lower government spending. As can be seen, due to the higher tax and lower GDP growth, our disposal income will be less, which result in the fall of stock price from 2007 to 2012. Since 2013, the tax rate had a huge decrease from 46.4% to 43.8%, hence, the stock price would be increased, which results in our real wealth began to over the expectation. From checking table 4.5, we can find during this period, we mainly invest into stock MCD.

Figure 4.10 United States tax rate (%) from 2007 to 2016



Model 3.1: Mean-VaR model (α = 0.01)

The wealth evolution with mean-VaR model (α =0.01) is presented in figure 4.11. It is apparent that the tendency is very similar with Markowitz model. During our analyzed period, the lowest real wealth is 904.32 USD, while the highest real return is 3037.43 USD. The fluctuation of real wealth path with mean-VaR decreased by 7% compared with Markowitz, hence, we articulate an idea that Mean-VaR model is more stable than Markowitz model. Observed from figure 4.10, there mainly are two phase that real return lower than expected return, first phase starts in September 2008 and ends in November 2011; the second stage is appeared from July 2016 till to the end of our investment horizon.

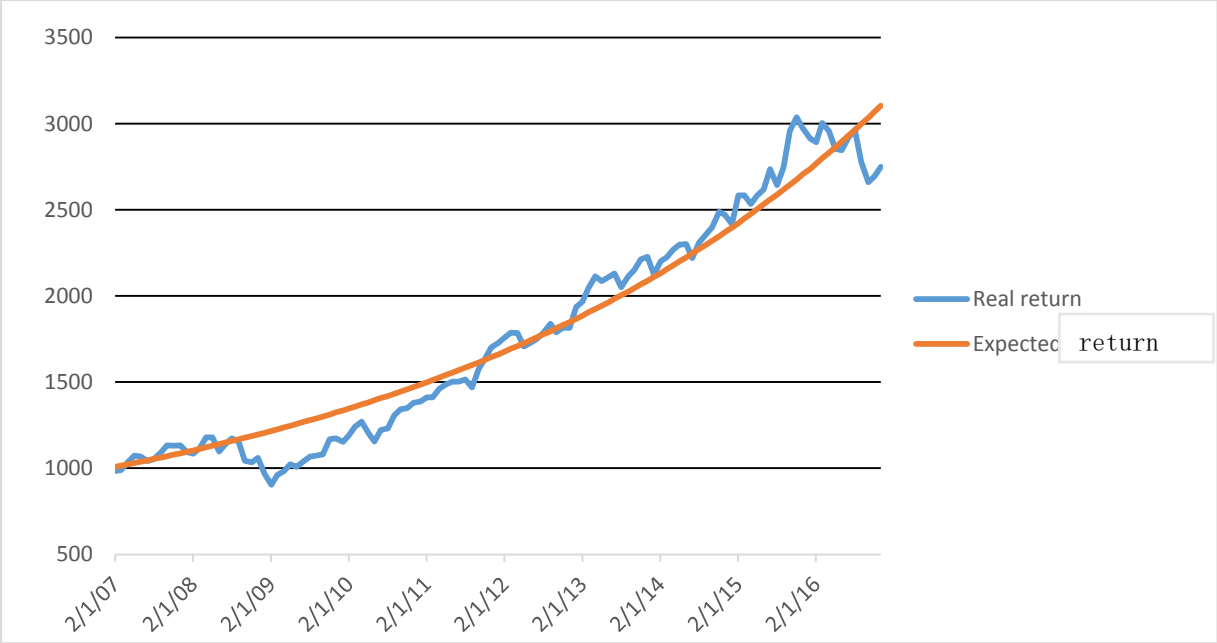


Figure 4.11 Wealth evolution with mean-VaR model (α = 0.01)

As we know, macroeconomic factors can influence the stock price. Here, we can give explanation about the real wealth lower than anticipation from 2008 to 2011, due to the higher interest rate. We can observed the change of interest rate, displayed by figure 4.12.

The relationship between stock price and interest rate is inverse, due to the net present value will decrease from the increase of interest rate, hence, stock price will decline with the increasing interest rate. Figure 4.11 illustrates there is extremely high interest rate of 3.06%, which caused the stock price fall, furthermore, affecting our wealth. Although the interest rate gradually decrease year by year from 2008 to 2011, it takes some times to recover the economy. Combining table 4.7, the optimal weights of each stock, we can observed we mainly invested

into stock HD, IBM, MCD, NKE, VZ and WMT from 2008 to 2011. Not all of these stocks perform well, therefore, our wealth may suffer loss from this reason as well.

As presented by table 4.7, we invest almost 80% of our principal into stocks HD and NKE since July 2016. It stands for higher risk. Although we can get higher return from higher risk, the some depreciation of stocks HD and NKE will lead to the relative huge decrease of our wealth. From checking the valuation ratio of HD[7], we can find the price to earnings ratio of stock HD is higher than industry average, as we know, the lower price to earnings ratio stands for less investment risk. Therefore, it is not very costly to invest too much into stock HD. This is the reason why our real wealth with some decrease from July 2016.

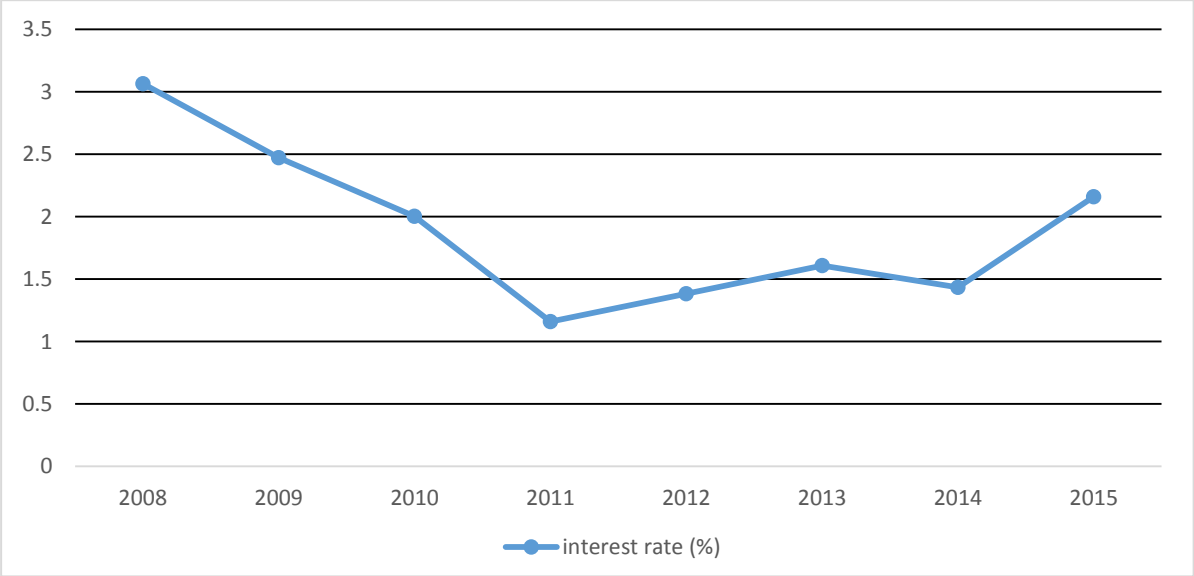


Figure 4.12 United States interest rate (%)

Model 3.2: Mean-VaR model (α = 0.15)

The visual representation of wealth path with mean-VaR (α = 0.15) is depicted in figure 4.13. We can find there is an upward tendency of real wealth evolution with bottom return of 905.44 USD and peak point of 3242.40 USD, which stands for the real wealth increased by 258% from February 2009 to November 2015. Observed from figure 4.12, the real wealth evolution path seems to be higher than expected wealth from January 2013 to the end of 2015.

Combining figure 4.8, the United States GDP growth rate, to analyze the performance of real wealth. There is a relative increase trend of GDP growth from 1.68% of financial year 2013 to 2.60% of 2015, which indicates during this period, the economy is relatively healthier in US,

[7] Source: Financial ratios. Available on website: http://financials.morningstar.com/valuation

the standard of living have improved, there is more spare cash for people can invest into stock market. Hence, the growth of GDP results in a more active stock market, we can get profit from the active stock market. This is the reason for the higher real wealth.

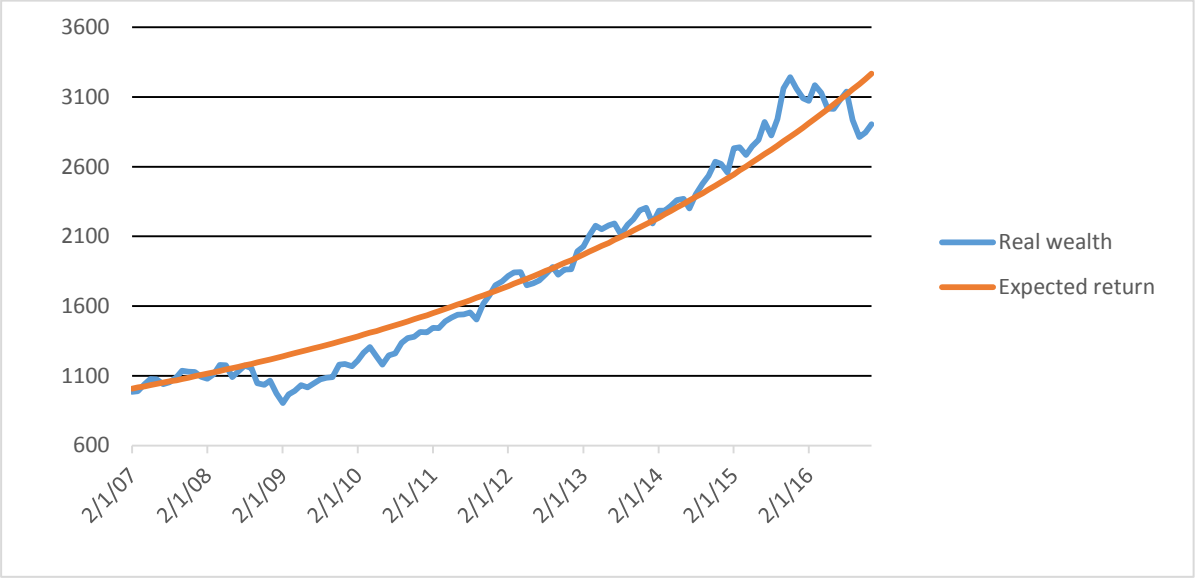


Figure 4.13 Wealth evolution with mean-VaR model ($\alpha = 0.15$)

We can find real wealth with 1% profitability level is less than it with 15% significance level on the whole. To analyze the results more precisely, we divide real wealth into two stage. First, from 2007 to 2009; then from 2010 to 2016.

The evolution path of both significance levels are almost to be seen as same in the first phase, however, we also can observe the real wealth are relatively higher due to the selection of higher confidence level ($\alpha = 0.01$). The selection of confidence level reflects, to a certain extent, the different risk preferences of financial institutions. Choosing a greater level of confidence represents the degree of risk aversion is high, therefore, hoping to get a better predictive result and making model is more accurate for extreme events. There is a huge impact of financial crisis for global economy, which results in a weaker stock market. Hence, model with 1% significance level relatively performs well during this period, it is the reason for the relative higher real wealth. However, the negative effect of financial crisis is too much, no matter what stocks we invest, the risk is high. This situation illustrates the reason why we lose our wealth under the selection of high confidence level.

The relation of real wealth between both different probability levels ($\alpha = 0.01$, $\alpha = 0.15$) is inverse since financial year 2010, real wealth with 15% probability level exceeds the 1%

significance level. It is caused by beginning of recovery of economy with sharply GDP growth from -2.78% of 2009 to 2.53% of 2010. It is obviously that the higher wealth can be obtained with the selection of greater risk ($\alpha = 0.15$) under the gradual stabilization of stock market. The visual representation of comparison of different significance level is presented in figure 4.14.

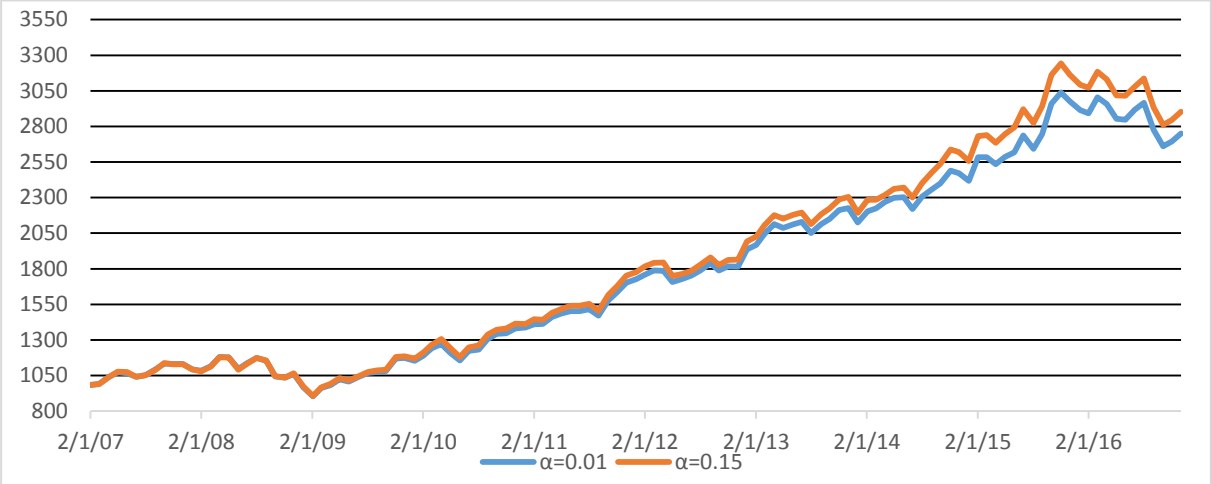
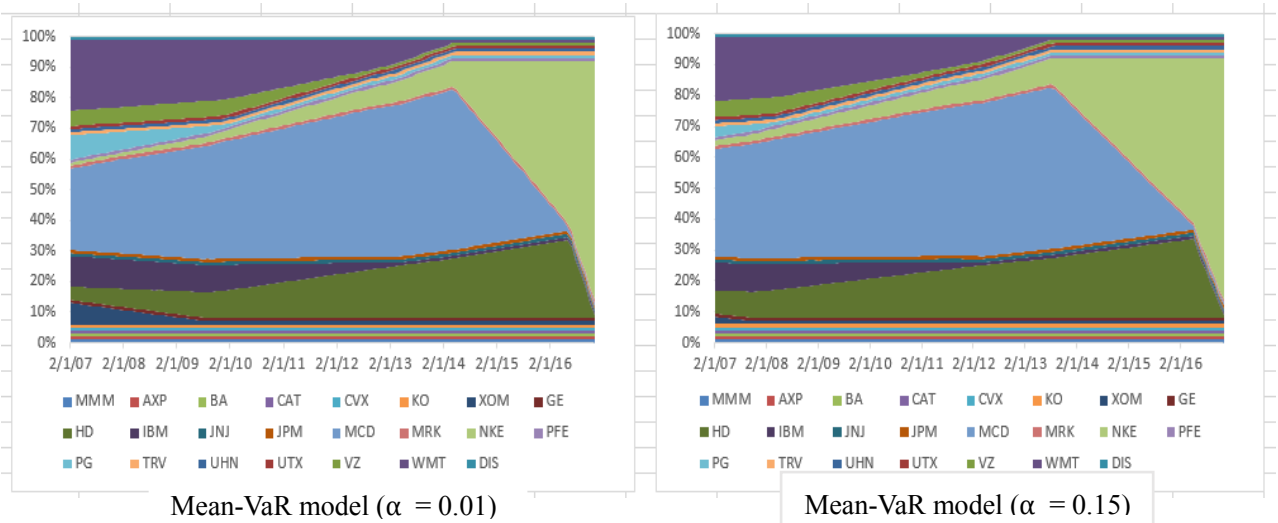


Figure 4.14 Comparison of different probability level

Displayed by figure 4.15, the comparison of optimal weights with the two particular setting, 1% and 15% probability level. The left graph represents the optimal share of each stock we should hold under confidence level of 99%; while the composition of each efficient portfolio with 85% confidence level is depicted in right graph. As can be seen, the weights of HD and MCD increased due to the change of significance level from 1% to 15%. By contrast, we should hold less share of stocks XOM, WMT, PG, IBM corresponding with the increase of probability level. Here, we concentrate on the analysis of stock MCD, whose share occupied the most during our investment horizon.

Figure 4.15 Optimal composition with different significance level



Mean-VaR model ($\alpha = 0.01$) Mean-VaR model ($\alpha = 0.15$)

For analysis, we back to table 4.7 and 4.8. If we start to invest principal of 1000 USD on February 3rd, 2007, we can find the optimal weights for each asset respectively are 7.09% of XOM, 4.23% of HD, 9.95% of IBM, 26.5% of MCD, 7.99 of PG, 5.09% of VZ and 23.15% of WMT. Therefore, the optimal value of VaR on the 1% significance level is 64.208 USD. It indicates our profit will be lower or equal of -64.208 USD, or, the forecast loss will be higher or equal to the amount of 64.208 with significance level of 1%. On the other side, with 15% probability, the optimal share for each assets are 2.37% of XOM, 7.54% of HD, 9.11% of IBM, 34.66% of MCD, 1.76% of NKE, 3.64% of PG, 5.20% of VZ and 20.72% of WMT. Hence, the value of VaR on the 15% significance level is 24.254 USD. It indicates our profit will be lower or equal of -24.254 USD, or, the forecast loss will be higher or equal to the amount of 24.254 with probability level of 15%.

4.4 Comparison of Models

In this subchapter, we will compare these different models based on the calculation of each portfolio introduced in previous sections. Therefore, we make three comparisons respectively about performance measure and wealth evolution

Table 4.9 Performance measure

	Markowitz model	Black model	Mean-VaR(0.01)	Mean-VaR(0.15)
Average real monthly return	0.91%	0.98%	0.91%	0.96%
Standard deviation	3.26%	4.16%	3.31%	3.40%
Sharpe ratio	21.56%	18.55%	21.12%	21.99%
Maximum drawdown	19.76%	27.10%	23.35%	23.11%
Final wealth	2771.06	2907.24	2750.32	2904.25

For performance measure, we mention the mean monthly return, the volatility of monthly return, the maximum drawdown and the Sharpe ratio.

From the measurement of mean monthly return, it is corresponding with final wealth value, obviously, we prefer to maximum it. As can be seen from table 4.8, the average real mean return for each model respectively are 0.91% of Markowitz model, 0.98% of Black's model, 0.91% of Mean-VaR model with 1% significance level, 0.96% of Mean-VaR model with 15% probability level.

From the return point of view, we inspect the results by final wealth, we observed that the highest final wealth appeared in Black's model, with 2907.24 USD, while the lowest final

wealth of 2750.32 USD was found in Mean-VaR model with 1% significance level. There was a difference of 156.92 USD. It indicates if we invest into stocks with principal of 1000 USD at the beginning of investment period, we can earn more by the selection of Black's model.

From the risk point of view, we measure the standard deviation of monthly wealth evolution. It is not difficult to find Markowitz model has the lowest risk. By contrast, Black's model has the highest standard deviation. As we know, short selling is allowed in Black's model, which means, we not only invest the initial principal but also cash obtained from other short sale into specific stocks. If the specific stocks experience some bad performance, we will suffer a huge loss. Hence, the standard deviation of Black's model is much higher.

As can be seen from maximum drawdown, we observe Markowitz model with the lowest maximum drawdown of 19.76%; Black's model with the highest maximum drawdown of 27.10%. The highest standard deviation and highest maximum drawdown both are appeared in Black's model, which indicates the risk of Black's model is so high. On the contrary, Markowitz model with the lowest standard deviation and maximum drawdown, it means, Markowitz model is relatively safer, due to it always try to minimize the risk and forbid short selling.

Considering about Sharpe ratio, who measure the relationship between risk and return. We can find black's model has the lowest Sharpe ratio, it indicates, the lower return will be provided under the same risk. By contrast, the highest Sharpe ratio of 21.99% appeared in model Mean-VaR with 15% probability level. In our thesis, we select ten years US government bond as the risk free rate.

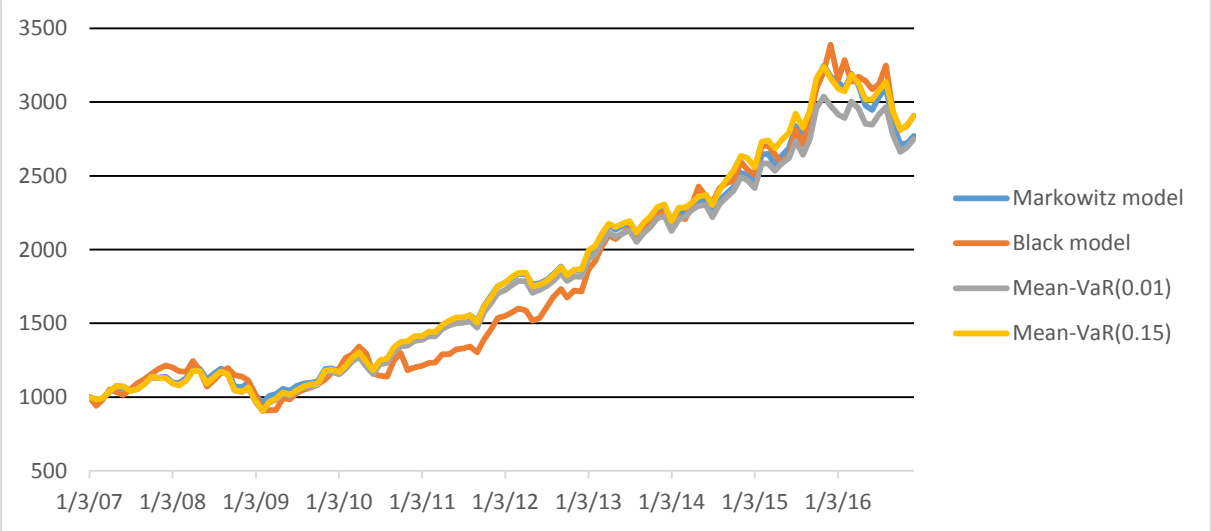


Figure 4.16 Comparison of real wealth path

As can be seen from figure 4.16, the visual representation of real wealth evolution among different models, there are almost similar tendencies. However, the volatility of black's model is relatively higher compared with others; the fluctuation of Mean-VaR with 1% significance level is relatively lower. Observed from figure 4.16, from financial year 2010 to 2013, the real wealth with black's model are obviously lower. Backing to table 4.5, the optimal composition of Black's model, we can find the weights for investing into stocks MMM and DIS are too much higher compared with other models, it may indicate invest into these two stocks are not so effective. The highest real wealth of these four models appeared in 2015, we can find during this period, we mainly invest our assets into stocks NKE, MCD and HD. Which indicates the investment is efficiency and can help us to make a profit.

As we know, macroeconomic indicators affect stock prices as well. If we want to maximize our wealth, it is necessary for us to understand the whole economy situation in United States. Before 2009, due to the negative influence of financial crisis, U.S. real gross domestic product (GDP) decreased by 6.3% in the fourth quarter of 2008, subsequently, down by 5.7% in the first quarter of 2009 as well. Which result in the Dow Jones index drop sharply, on October 9th, the index fell below 9000 points. Due to all of stocks come from DJIA, the negative growth of index has a direct influence on our investment, hence, we experienced a bad situation during this period. Since 2010, the economy began to recovery, due to the growth of GDP and decrease of unemployment rate, New York Stock Exchange tends to active, there is a good opportunity for us to invest into stock market. We also can discover, the number of holding stocks has a decreased tendency. As we know, diversification can help us to reduce the risk of investment. For period 2007 to 2009, the economy was recession and stock market was weaker, if we try to do some investments in this period, we have to concentrate on risk more, hence, we invest into many stocks to reduce our whole risk. With the economy recovery and better stock market, we can pay more attention to return, therefore, we reduce the total number of holding stocks and invest into some specific stock with higher return rate. At last, we observe the whole tendency of real wealth path followed by the volatility of DJIA.

Furthermore, for better analysis, we combine table 4.9 and figure 4.16. It is easy to find Markowitz model is more stable among these four, due to the relative lower volatility and lowest standard deviation and maximum drawdown. It indicates we suffer less loss. And Sharpe ratio of Markowitz model ranks the second, it indicates the same return can obtain by suffering less risk by the selection of Markowitz model.

5. Conclusion

The reason for stock portfolio management is to maximize investor's utility, that is, the greatest satisfaction can bring to investors with the combination of risk and return of stock portfolios. Therefore, there are two reasons to build a stock portfolio: one is to reduce the investment risk of securities; the second is to maximize the investment return of securities. The nature of investing is trade – off between the uncertainty return and risk. Therefore, it is necessary for us to construct the optimal composition and set efficient frontier.

In order to achieve the goal of our thesis, which is to make selection and determination about stock portfolios from trading off return and risk for assets allocation. We firstly should understand the basic information about portfolio management, investment decision making and investment process of portfolios, which are introduced in chapter 2. Then we introduce the theoretical knowledge of several alternative strategies and their different objective functions and constraints in chapter 3.

According to the analysis of chapter 4, we can state the performance of each alternative model. From the return point of view, as displayed by table 4.8, the average real mean return for each model respectively are 0.91% of Markowitz model, 0.98% of Black's model, 0.91% of Mean-VaR model with 1% significance level, 0.96% of Mean-VaR model with 15% probability level. Then, we check the final wealth of each strategy, we can find the monthly return rate is corresponding with final wealth value, obviously, we prefer to maximum it. Hence, from return point, Black's model is better.

From the risk point of view, we measure the standard deviation of monthly wealth evolution. It is not difficult to find Markowitz model has the lowest risk 3.26%. By contrast, Black's model has the highest standard deviation. As we know, short selling is permitted in Black's model, which means, we not only invest the initial principal but also cash obtained from other short sale into specific stocks. If the specific stocks experience some bad performance, we will suffer a huge loss. Under the consideration of risk, Black's model is not satisfied.

Considering about risk and return together, we can check the performance of Sharpe ratio. It is not difficult to find black's model has the lowest Sharpe ratio, it indicates, the lower return will be provided under the same risk. By contrast, Sharpe ratio of Markowitz model ranks the

second, it indicates the same return can obtain by suffering less risk by the selection of Markowitz model.

We can make a conclusion about that Markowitz model seems like to be more profitable and safer than other models. Thus, if we want to invest in New York Stock Exchange, we can choose Markowitz model as our investment strategy, then, in dependence on the risk profile, we can select the most optimal composition for the assets allocation.

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List of Abbreviations

MMM: 3M

AXP: American Express

BA: Boeing

CAT: Caterpillar

CVX: Chevron

KO: Coca-Cola

ED: Consolidated Edison

DUK: Duke Energy

EXC: Exelon

XOM: ExxonMobil

FDX: FedEx

GE: General Electric

HD: The Home Depot

SO: The southern company

IBM: IBM

JNJ: Johnson & Johnson

IPM: JPMorgan Chase

MCD: McDonald's

MRK: Merck

NKE: Nike

NSC: Norfolk Southern

PFE: Pfizer

PG: Procter & Gamble

PCG: PG & E Corporation

Public Service Enterprise

PEG: Public Service Enterprise

R: Ryder system

LUV: Southwest Airlines

TRV: Travelers

UNH: UnitedHealth Group

UTX: United Technologies

UNP: Union Pacific Corporation

VZ: Verizon

WMT: Wal-Mart

DIS: Walt Disney

SR: Sharpe ratio

Max. DD: Maximum drawdown

VaR: Value at Risk

MPT: Modern portfolio theory

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Student's name and surname

List of Annexes

Annex 1: Historical monthly adjust close price (Unit: USD)

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS
1/3/07	56.89	49.24	69.18	48.17	50.78	17.77	…	39.29	46.73	53.61	21.98	37.36	29.97
2/1/07	57.08	48.10	67.66	48.44	48.08	17.33	…	39.21	46.66	51.94	21.34	37.84	29.18
3/1/07	58.89	47.70	68.94	50.39	51.90	17.94	…	40.20	47.39	51.43	21.63	36.95	29.34
4/2/07	63.77	51.45	72.11	54.83	54.59	19.51	…	42.01	47.47	53.12	22.02	37.71	29.81
5/1/07	68.15	55.10	78.29	59.33	57.60	19.81	…	42.07	49.00	56.04	25.10	37.64	30.20
6/1/07	67.25	51.88	74.84	59.12	59.54	19.69	…	41.77	45.75	56.34	23.74	38.04	29.92
7/2/07	68.90	49.76	80.50	59.75	60.26	19.61	…	39.64	43.33	57.96	24.82	36.33	28.92
8/1/07	70.88	49.83	75.51	57.45	62.47	20.24	…	39.46	44.74	59.54	24.39	34.67	29.45
9/4/07	72.90	50.47	81.99	59.47	66.61	21.76	…	39.53	43.33	64.21	25.78	34.69	30.14
10/1/07	67.28	51.94	76.99	56.84	65.14	23.39	…	41.00	43.97	61.11	27.08	35.93	30.35
11/1/07	65.25	50.26	72.53	54.77	62.90	23.64	…	41.70	49.20	59.91	25.40	38.07	29.05
12/3/07	66.08	44.33	68.54	55.28	66.88	23.36	…	42.47	52.07	61.33	25.68	37.94	28.60
1/2/08	62.42	42.01	65.19	54.38	59.66	22.46	…	37.97	45.48	58.71	23.06	40.50	26.44
2/1/08	61.82	36.17	65.20	55.41	62.55	22.26	…	36.64	41.58	56.74	21.57	39.59	28.71
3/3/08	62.42	37.39	58.57	59.98	61.61	23.32	…	38.01	30.77	55.38	21.64	42.25	27.80
4/1/08	60.64	41.22	66.83	63.01	69.40	22.56	…	40.03	29.22	58.32	23.22	46.50	28.73
5/1/08	61.56	39.79	65.49	63.60	72.04	21.94	…	39.56	30.63	57.42	23.21	46.51	29.77
6/2/08	55.23	32.34	52.00	56.81	72.03	20.05	…	34.68	23.50	49.87	21.36	45.27	27.64
7/1/08	55.87	32.00	48.35	53.82	61.44	19.86	…	35.25	25.14	51.71	20.79	47.22	26.89
8/1/08	57.23	34.21	52.19	54.76	63.19	20.08	…	35.29	27.26	53.27	21.45	47.77	28.66
9/2/08	54.60	30.55	45.66	46.14	60.38	20.54	…	36.37	22.73	48.78	19.60	48.43	27.19
10/1/08	51.39	23.83	41.73	29.87	54.61	17.11	…	34.24	21.25	44.63	18.42	45.13	22.95
11/3/08	53.93	20.20	34.19	32.05	58.34	18.36	…	35.12	18.81	39.71	20.27	45.19	19.95
12/1/08	46.36	16.08	34.23	34.93	54.62	17.73	…	36.62	23.82	43.86	21.04	45.53	20.40
1/2/09	43.34	14.62	33.94	24.38	52.07	16.74	…	31.31	25.37	39.27	18.81	38.27	18.60
2/2/09	37.03	10.54	25.47	19.45	45.24	16.00	…	29.29	17.59	33.69	17.97	39.99	15.08
3/2/09	40.50	11.91	28.83	22.10	50.11	17.40	…	33.22	18.77	35.47	19.02	42.55	16.33
4/1/09	46.92	22.34	32.45	28.49	49.26	17.04	…	33.63	21.09	40.30	19.38	41.17	19.69
5/1/09	46.92	22.01	36.69	28.39	50.17	19.46	…	33.24	23.85	43.73	18.69	40.85	21.78
6/1/09	49.38	20.74	34.77	26.45	49.85	19.16	…	33.78	22.40	43.19	19.63	39.78	20.98
7/1/09	57.94	25.28	35.11	35.72	52.27	19.90	…	35.45	25.16	45.28	20.81	40.96	22.59
8/3/09	59.67	30.18	41.03	36.73	53.15	19.47	…	41.50	25.11	49.68	20.14	42.00	23.42
9/1/09	61.08	30.42	44.73	41.61	53.52	21.62	…	40.76	22.45	50.99	19.64	40.53	24.69
10/1/09	60.89	31.26	39.48	44.96	58.17	21.46	…	41.23	23.27	51.43	19.50	41.02	24.61
11/2/09	64.51	37.53	43.67	47.68	59.83	23.19	…	43.38	25.71	56.58	20.74	45.04	27.17
12/1/09	68.87	36.35	45.11	46.54	59.03	23.11	…	41.56	27.33	58.41	21.84	44.36	29.33
1/4/10	67.05	33.94	50.50	42.95	55.29	21.99	…	42.23	29.59	56.79	19.67	44.34	26.88
2/1/10	67.21	34.41	52.99	46.90	55.96	21.37	…	43.83	30.36	58.14	19.35	44.87	28.42
3/1/10	70.07	37.34	60.92	51.67	58.69	22.48	…	45.24	29.30	62.34	20.74	46.40	31.75
4/1/10	74.35	41.74	60.77	56.33	63.03	21.85	…	42.55	27.20	63.48	19.62	44.76	33.51

5/3/10	66.92	36.08	54.16	50.26	57.71	21.01	…	41.49	26.09	57.40	18.69	42.44	30.40
6/1/10	66.65	36.09	52.95	49.69	53.01	20.66	…	41.61	25.60	55.30	19.03	40.35	28.65
7/1/10	72.18	40.58	57.50	58.09	59.54	22.71	…	42.63	27.44	60.57	21.42	42.97	30.64
8/2/10	66.69	36.25	51.90	54.26	58.41	23.03	…	41.39	28.59	55.89	21.77	42.33	29.60
9/1/10	73.62	38.21	56.49	65.52	63.91	24.30	…	44.34	31.76	61.05	24.02	45.19	30.11
10/1/10	71.51	37.87	59.97	65.82	65.13	25.47	…	46.98	32.61	64.08	24.30	45.73	32.86
11/1/10	71.75	39.48	54.47	70.84	64.39	26.41	…	45.95	33.04	64.88	23.94	45.67	33.21
12/1/10	73.73	39.20	55.74	78.43	72.57	27.50	…	47.72	32.77	67.86	26.76	45.78	34.49
1/3/11	75.12	39.79	59.34	81.61	75.50	26.28	…	48.20	37.26	70.08	26.99	47.60	35.74
2/1/11	79.28	39.96	61.86	86.59	83.13	26.73	…	51.34	38.65	72.37	27.98	44.13	40.22
3/1/11	80.37	41.46	63.51	93.68	86.13	27.94	…	51.27	41.14	73.34	29.21	44.50	39.62
4/1/11	83.56	45.19	68.54	97.50	87.69	28.41	…	54.54	44.81	77.61	29.00	47.00	39.63
5/2/11	81.60	47.51	67.39	89.38	84.71	28.14	…	53.51	44.55	76.46	28.35	47.52	38.28
6/1/11	82.01	47.78	63.85	89.94	83.04	28.55	…	50.66	47.10	77.10	28.58	45.73	35.90
7/1/11	75.34	46.25	60.86	83.81	83.99	28.85	…	47.84	45.32	72.16	27.44	45.36	35.51
8/1/11	72.23	45.94	58.13	77.20	80.45	29.89	…	43.79	43.40	65.11	28.13	46.10	31.32
9/1/11	62.49	41.49	52.61	62.64	75.36	28.86	…	42.64	42.26	61.70	28.62	44.99	27.73
10/3/11	68.78	46.98	57.20	80.58	85.50	29.18	…	51.06	43.98	68.38	29.16	49.16	32.07
11/1/11	71.04	44.58	60.11	83.50	84.35	28.92	…	49.23	44.69	67.58	29.75	51.05	32.97
12/1/11	71.64	43.77	64.18	77.29	87.29	30.10	…	52.17	46.60	64.48	31.63	52.12	35.06
1/3/12	76.01	46.70	64.91	93.50	84.60	29.05	…	51.40	47.62	69.12	30.08	53.52	36.37
2/1/12	77.31	49.27	65.97	97.86	90.20	30.06	…	51.11	51.26	74.42	30.44	51.53	39.26
3/1/12	78.73	53.90	65.46	91.27	88.63	32.07	…	52.57	54.35	73.59	30.54	53.74	40.93
4/2/12	78.86	56.28	67.60	88.43	88.09	33.08	…	57.12	51.78	72.44	32.68	51.72	40.31
5/1/12	75.01	52.18	61.63	75.40	82.00	32.39	…	55.49	51.43	66.17	33.70	58.18	42.74
6/1/12	79.62	54.41	65.78	73.06	88.00	34.12	…	57.12	54.15	67.44	35.96	61.63	45.35
7/2/12	81.07	54.12	65.44	72.93	91.40	35.26	…	56.06	47.29	66.47	36.94	65.80	45.94
8/1/12	82.81	54.68	63.59	73.89	94.30	32.64	…	57.93	50.26	71.79	35.14	64.52	46.25
9/4/12	82.65	53.33	61.99	74.51	98.00	33.33	…	61.52	51.50	70.39	37.29	65.59	48.88
10/1/12	78.34	52.68	62.74	73.89	92.69	32.67	…	63.93	52.04	70.27	36.94	66.68	45.93
11/1/12	81.87	52.61	66.56	74.26	89.63	33.55	…	63.83	50.55	72.53	36.50	64.01	46.43
12/3/12	83.58	54.10	67.53	78.52	91.70	32.07	…	65.13	50.61	74.25	35.80	60.98	47.27
1/2/13	90.52	55.54	66.20	86.21	97.65	32.94	…	71.15	51.51	79.28	36.50	62.51	51.15
2/1/13	94.20	58.70	69.36	80.94	100.12	34.25	…	72.93	49.87	82.47	38.95	63.26	51.82
3/1/13	96.29	63.71	77.43	76.21	101.55	36.03	…	76.78	53.59	85.10	41.14	67.31	53.92
4/1/13	94.84	64.80	82.44	74.67	104.27	37.72	…	77.90	56.13	83.15	45.60	69.91	59.66
5/1/13	100.45	71.72	89.76	75.66	105.75	35.63	…	76.35	58.66	86.92	41.01	67.72	59.88
6/3/13	99.61	70.82	92.86	72.75	101.95	35.99	…	73.33	61.60	85.12	42.58	67.40	59.95
7/1/13	106.97	70.09	95.28	73.63	108.46	35.96	…	76.67	68.54	96.69	42.28	70.52	61.37
8/1/13	104.04	68.32	94.64	73.30	104.61	34.26	…	73.32	67.49	92.14	40.48	66.44	57.75
9/3/13	109.38	71.75	107.00	74.06	105.54	34.23	…	78.27	67.62	99.25	39.88	67.33	61.22
10/1/13	115.27	77.96	118.84	74.54	104.20	35.76	…	79.68	64.46	97.81	43.65	69.87	65.11
11/1/13	122.89	81.77	122.70	75.65	107.25	36.58	…	83.78	70.33	102.61	42.88	73.75	66.97

12/2/13	129.09	86.47	124.75	81.21	109.41	37.59	…	84.08	71.38	105.33	42.46	72.05	73.42
1/2/14	117.99	81.24	114.48	84.53	97.78	34.42	…	75.48	68.52	105.54	41.94	68.38	69.78
2/3/14	124.83	87.22	118.49	87.28	101.92	34.76	…	77.85	73.25	108.88	41.56	68.40	77.66
3/3/14	125.69	86.02	115.34	89.44	105.08	35.46	…	79.50	78.00	108.72	41.55	70.43	76.95
4/1/14	128.87	83.75	118.59	95.43	110.92	37.42	…	84.61	71.39	110.10	41.27	73.46	76.25
5/1/14	132.88	87.65	125.01	92.56	109.44	37.53	…	87.30	75.76	108.68	44.13	71.19	80.74
6/2/14	133.53	90.88	117.60	98.39	116.35	39.15	…	88.39	78.14	107.96	43.22	69.61	82.40
7/1/14	131.34	84.53	111.36	91.79	115.18	36.31	…	84.15	77.47	98.33	45.01	68.23	82.53
8/1/14	135.04	86.02	117.91	99.37	116.35	38.56	…	88.99	82.85	101.54	44.48	70.47	86.38
9/2/14	132.86	84.09	118.45	90.23	107.25	39.71	…	88.78	82.79	99.31	44.63	71.38	85.56
10/1/14	144.20	86.66	116.15	93.10	107.81	38.99	…	95.27	91.20	100.62	45.36	71.19	87.82
11/3/14	150.93	89.04	125.67	92.36	98.76	42.02	…	98.72	94.68	104.09	45.67	81.71	88.91
12/1/14	154.92	89.64	121.57	84.03	101.76	39.57	…	100.56	97.40	108.74	42.23	80.61	91.67
1/2/15	153.01	77.97	135.97	74.02	93.01	38.59	…	97.69	102.37	108.53	41.75	79.76	88.53
2/2/15	159.99	78.84	141.96	76.73	97.73	40.59	…	102.08	109.49	115.89	45.17	78.78	101.30
3/2/15	156.48	75.73	141.24	74.07	96.17	38.32	…	103.26	114.36	111.42	44.42	77.66	102.09
4/1/15	148.36	75.09	134.90	81.08	101.74	38.33	…	96.56	107.70	108.14	46.59	73.70	105.81
5/1/15	151.87	77.29	133.09	79.62	95.30	38.71	…	96.56	116.21	112.00	45.67	70.57	107.42
6/1/15	147.30	75.63	131.38	79.16	89.25	37.38	…	92.88	118.44	106.03	43.05	67.40	111.09
7/1/15	144.48	74.01	136.55	74.06	81.86	39.15	…	101.97	117.86	95.88	43.73	68.39	117.47
8/3/15	136.64	74.66	124.56	72.00	75.88	37.47	…	95.65	112.33	88.14	43.00	61.93	99.73
9/1/15	136.28	72.14	124.82	61.56	73.90	38.56	…	96.23	113.11	85.62	40.66	62.03	100.05
10/1/15	151.12	71.56	141.14	69.51	85.14	40.71	…	109.15	114.84	94.68	44.38	54.76	111.34
11/2/15	151.51	69.98	139.50	69.19	86.60	41.28	…	110.78	109.90	93.01	43.03	56.29	111.08
12/1/15	145.76	67.94	138.67	64.72	85.31	41.61	…	109.71	115.20	93.03	43.76	59.14	103.52
1/4/16	146.11	52.49	115.21	60.02	82.00	41.57	…	104.05	112.77	84.91	47.89	64.02	94.40
2/1/16	152.89	54.53	114.39	65.28	80.13	41.77	…	104.52	116.63	94.26	48.62	64.00	94.11
3/1/16	162.40	60.24	122.87	73.81	91.61	45.28	…	114.08	126.75	97.65	51.83	66.56	97.84
4/1/16	163.13	64.50	130.48	75.68	98.13	43.73	…	107.43	129.48	101.82	49.33	64.99	101.73
5/2/16	165.14	64.83	123.10	70.60	98.02	43.53	…	111.57	131.44	98.77	49.30	69.29	97.75
6/1/16	171.81	60.20	126.73	73.82	101.73	44.59	…	117.05	139.47	100.71	54.08	71.48	96.37
7/1/16	174.99	63.86	130.43	81.36	99.45	42.91	…	114.27	141.45	105.71	54.21	71.43	95.22
8/1/16	176.95	64.97	127.37	80.56	98.64	42.72	…	116.72	134.39	105.15	51.19	70.41	93.74
9/1/16	173.97	63.45	129.62	87.27	100.93	41.97	…	113.27	138.93	100.38	50.85	71.08	92.15
10/3/16	163.18	66.14	140.14	82.77	102.73	42.04	…	106.97	140.25	100.97	47.59	69.01	91.98
11/1/16	170.63	71.73	149.28	94.77	110.50	40.35	…	112.08	157.73	107.08	49.37	69.42	98.36
12/1/16	177.41	73.77	154.35	91.98	116.58	41.46	…	121.75	159.45	108.97	52.82	68.62	104.22

Annex 2: Historical monthly return (Unit: USD)

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS
2/1/07	0.3%	-2.3%	-2.2%	0.6%	-5.5%	-2.5%	…	-0.2%	-0.2%	-3.2%	-3.0%	1.3%	-2.7%
3/1/07	3.1%	-0.8%	1.9%	4.0%	7.6%	3.5%	…	2.5%	1.6%	-1.0%	1.4%	-2.4%	0.5%
4/2/07	8.0%	7.6%	4.5%	8.4%	5.0%	8.4%	…	4.4%	0.2%	3.2%	1.8%	2.0%	1.6%
5/1/07	6.6%	6.9%	8.2%	7.9%	5.4%	1.5%	…	0.1%	3.2%	5.4%	13.1%	-0.2%	1.3%
6/1/07	-1.3%	-6.0%	-4.5%	-0.4%	3.3%	-0.6%	…	-0.7%	-6.9%	0.5%	-5.6%	1.1%	-0.9%
7/2/07	2.4%	-4.2%	7.3%	1.1%	1.2%	-0.4%	…	-5.2%	-5.4%	2.8%	4.4%	-4.6%	-3.4%
8/1/07	2.8%	0.1%	-6.4%	-3.9%	3.6%	3.2%	…	-0.5%	3.2%	2.7%	-1.8%	-4.7%	1.8%
9/4/07	2.8%	1.3%	8.2%	3.5%	6.4%	7.2%	…	0.2%	-3.2%	7.5%	5.6%	0.0%	2.3%
10/1/07	-8.0%	2.9%	-6.3%	-4.5%	-2.2%	7.2%	…	3.6%	1.5%	-5.0%	4.9%	3.5%	0.7%
11/1/07	-3.1%	-3.3%	-6.0%	-3.7%	-3.5%	1.1%	…	1.7%	11.2%	-2.0%	-6.4%	5.8%	-4.4%
12/3/07	1.3%	-12.6%	-5.6%	0.9%	6.1%	-1.2%	…	1.8%	5.7%	2.3%	1.1%	-0.3%	-1.6%
1/2/08	-5.7%	-5.4%	-5.0%	-1.6%	-11.4%	-3.9%	…	-11.2%	-13.5%	-4.4%	-10.8%	6.5%	-7.9%
2/1/08	-1.0%	-15.0%	0.0%	1.9%	4.7%	-0.9%	…	-3.6%	-9.0%	-3.4%	-6.7%	-2.3%	8.3%
3/3/08	1.0%	3.3%	-10.7%	7.9%	-1.5%	4.7%	…	3.7%	-30.1%	-2.4%	0.4%	6.5%	-3.2%
4/1/08	-2.9%	9.8%	13.2%	4.9%	11.9%	-3.3%	…	5.2%	-5.2%	5.2%	7.0%	9.6%	3.3%
5/1/08	1.5%	-3.5%	-2.0%	0.9%	3.7%	-2.8%	…	-1.2%	4.7%	-1.6%	0.0%	0.0%	3.5%
6/2/08	-10.8%	-20.7%	-23.1%	-11.3%	0.0%	-9.0%	…	-13.2%	-26.5%	-14.1%	-8.3%	-2.7%	-7.4%
7/1/08	1.1%	-1.0%	-7.3%	-5.4%	-15.9%	-0.9%	…	1.6%	6.7%	3.6%	-2.7%	4.2%	-2.8%
8/1/08	2.4%	6.7%	7.6%	1.7%	2.8%	1.1%	…	0.1%	8.1%	3.0%	3.1%	1.2%	6.4%
9/2/08	-4.7%	-11.3%	-13.4%	-17.1%	-4.6%	2.3%	…	3.0%	-18.2%	-8.8%	-9.0%	1.4%	-5.3%
10/1/08	-6.0%	-24.8%	-9.0%	-43.5%	-10.0%	-18.2%	…	-6.0%	-6.8%	-8.9%	-6.2%	-7.1%	-16.9%
11/3/08	4.8%	-16.5%	-19.9%	7.0%	6.6%	7.0%	…	2.6%	-12.2%	-11.7%	9.6%	0.1%	-14.0%
12/1/08	-15.1%	-22.8%	0.1%	8.6%	-6.6%	-3.5%	…	4.2%	23.6%	9.9%	3.8%	0.8%	2.2%
1/2/09	-6.7%	-9.5%	-0.8%	-35.9%	-4.8%	-5.8%	…	-15.7%	6.3%	-11.1%	-11.2%	-17.4%	-9.3%
2/2/09	-15.7%	-32.7%	-28.7%	-22.6%	-14.1%	-4.5%	…	-6.7%	-36.6%	-15.3%	-4.6%	4.4%	-21.0%
3/2/09	9.0%	12.2%	12.4%	12.8%	10.2%	8.4%	…	12.6%	6.5%	5.1%	5.7%	6.2%	8.0%
4/1/09	14.7%	62.9%	11.8%	25.4%	-1.7%	-2.1%	…	1.2%	11.7%	12.8%	1.9%	-3.3%	18.7%
5/1/09	0.0%	-1.5%	12.3%	-0.3%	1.8%	13.3%	…	-1.2%	12.3%	8.2%	-3.6%	-0.8%	10.1%
6/1/09	5.1%	-5.9%	-5.4%	-7.1%	-0.6%	-1.6%	…	1.6%	-6.3%	-1.2%	4.9%	-2.6%	-3.7%
7/1/09	16.0%	19.8%	1.0%	30.0%	4.7%	3.8%	…	4.8%	11.6%	4.7%	5.8%	2.9%	7.4%
8/3/09	2.9%	17.7%	15.6%	2.8%	1.7%	-2.2%	…	15.8%	-0.2%	9.3%	-3.3%	2.5%	3.6%
9/1/09	2.3%	0.8%	8.6%	12.5%	0.7%	10.4%	…	-1.8%	-11.2%	2.6%	-2.5%	-3.6%	5.3%
10/1/09	-0.3%	2.7%	-12.5%	7.7%	8.3%	-0.7%	…	1.1%	3.6%	0.8%	-0.7%	1.2%	-0.3%
11/2/09	5.8%	18.3%	10.1%	5.9%	2.8%	7.8%	…	5.1%	10.0%	9.6%	6.1%	9.4%	9.9%
12/1/09	6.5%	-3.2%	3.2%	-2.4%	-1.4%	-0.4%	…	-4.3%	6.1%	3.2%	5.2%	-1.5%	7.6%
1/4/10	-2.7%	-6.9%	11.3%	-8.0%	-6.5%	-4.9%	…	1.6%	7.9%	-2.8%	-10.4%	0.0%	-8.7%
2/1/10	0.2%	1.4%	4.8%	8.8%	1.2%	-2.9%	…	3.7%	2.6%	2.4%	-1.7%	1.2%	5.6%
3/1/10	4.2%	8.2%	13.9%	9.7%	4.8%	5.0%	…	3.1%	-3.6%	7.0%	7.0%	3.4%	11.1%
4/1/10	5.9%	11.1%	-0.2%	8.6%	7.1%	-2.9%	…	-6.1%	-7.4%	1.8%	-5.5%	-3.6%	5.4%

5/3/10	-10.5%	-14.6%	-11.5%	-11.4%	-8.8%	-3.9%	…	-2.5%	-4.2%	-10.1%	-4.9%	-5.3%	-9.7%
6/1/10	-0.4%	0.0%	-2.3%	-1.1%	-8.5%	-1.7%	…	0.3%	-1.9%	-3.7%	1.8%	-5.1%	-5.9%
7/1/10	8.0%	11.7%	8.2%	15.6%	11.6%	9.5%	…	2.4%	7.0%	9.1%	11.9%	6.3%	6.7%
8/2/10	-7.9%	-11.3%	-10.3%	-6.8%	-1.9%	1.4%	…	-2.9%	4.1%	-8.0%	1.6%	-1.5%	-3.5%
9/1/10	9.9%	5.3%	8.5%	18.9%	9.0%	5.4%	…	6.9%	10.5%	8.8%	9.9%	6.5%	1.7%
10/1/10	-2.9%	-0.9%	6.0%	0.4%	1.9%	4.7%	…	5.8%	2.6%	4.9%	1.1%	1.2%	8.8%
11/1/10	0.3%	4.2%	-9.6%	7.4%	-1.1%	3.7%	…	-2.2%	1.3%	1.2%	-1.5%	-0.1%	1.0%
12/1/10	2.7%	-0.7%	2.3%	10.2%	12.0%	4.0%	…	3.8%	-0.8%	4.5%	11.1%	0.3%	3.8%
1/3/11	1.9%	1.5%	6.3%	4.0%	4.0%	-4.5%	…	1.0%	12.8%	3.2%	0.9%	3.9%	3.6%
2/1/11	5.4%	0.4%	4.2%	5.9%	9.6%	1.7%	…	6.3%	3.7%	3.2%	3.6%	-7.6%	11.8%
3/1/11	1.4%	3.7%	2.6%	7.9%	3.5%	4.4%	…	-0.1%	6.3%	1.3%	4.3%	0.8%	-1.5%
4/1/11	3.9%	8.6%	7.6%	4.0%	1.8%	1.7%	…	6.2%	8.5%	5.7%	-0.7%	5.5%	0.0%
5/2/11	-2.4%	5.0%	-1.7%	-8.7%	-3.5%	-1.0%	…	-1.9%	-0.6%	-1.5%	-2.3%	1.1%	-3.5%
6/1/11	0.5%	0.6%	-5.4%	0.6%	-2.0%	1.4%	…	-5.5%	5.6%	0.8%	0.8%	-3.8%	-6.4%
7/1/11	-8.5%	-3.3%	-4.8%	-7.1%	1.1%	1.1%	…	-5.7%	-3.9%	-6.6%	-4.1%	-0.8%	-1.1%
8/1/11	-4.2%	-0.7%	-4.6%	-8.2%	-4.3%	3.5%	…	-8.9%	-4.3%	-10.3%	2.5%	1.6%	-12.6%
9/1/11	-14.5%	-10.2%	-10.0%	-20.9%	-6.5%	-3.5%	…	-2.6%	-2.6%	-5.4%	1.7%	-2.5%	-12.2%
10/3/11	9.6%	12.4%	8.4%	25.2%	12.6%	1.1%	…	18.0%	4.0%	10.3%	1.9%	8.9%	14.5%
11/1/11	3.2%	-5.2%	4.9%	3.6%	-1.4%	-0.9%	…	-3.7%	1.6%	-1.2%	2.0%	3.8%	2.7%
12/1/11	0.8%	-1.8%	6.6%	-7.7%	3.4%	4.0%	…	5.8%	4.2%	-4.7%	6.1%	2.1%	6.2%
1/3/12	5.9%	6.5%	1.1%	19.0%	-3.1%	-3.5%	…	-1.5%	2.2%	6.9%	-5.0%	2.6%	3.7%
2/1/12	1.7%	5.3%	1.6%	4.6%	6.4%	3.4%	…	-0.6%	7.4%	7.4%	1.2%	-3.8%	7.6%
3/1/12	1.8%	9.0%	-0.8%	-7.0%	-1.8%	6.5%	…	2.8%	5.9%	-1.1%	0.3%	4.2%	4.2%
4/2/12	0.2%	4.3%	3.2%	-3.2%	-0.6%	3.1%	…	8.3%	-4.8%	-1.6%	6.8%	-3.8%	-1.5%
5/1/12	-5.0%	-7.6%	-9.2%	-15.9%	-7.2%	-2.1%	…	-2.9%	-0.7%	-9.0%	3.1%	11.8%	5.9%
6/1/12	6.0%	4.2%	6.5%	-3.1%	7.1%	5.2%	…	2.9%	5.1%	1.9%	6.5%	5.8%	5.9%
7/2/12	1.8%	-0.5%	-0.5%	-0.2%	3.8%	3.3%	…	-1.9%	-13.5%	-1.5%	2.7%	6.5%	1.3%
8/1/12	2.1%	1.0%	-2.9%	1.3%	3.1%	-7.7%	…	3.3%	6.1%	7.7%	-5.0%	-2.0%	0.7%
9/4/12	-0.2%	-2.5%	-2.6%	0.8%	3.8%	2.1%	…	6.0%	2.4%	-2.0%	5.9%	1.6%	5.5%
10/1/12	-5.4%	-1.2%	1.2%	-0.8%	-5.6%	-2.0%	…	3.9%	1.1%	-0.2%	-1.0%	1.6%	-6.2%
11/1/12	4.4%	-0.1%	5.9%	0.5%	-3.4%	2.7%	…	-0.2%	-2.9%	3.2%	-1.2%	-4.1%	1.1%
12/3/12	2.1%	2.8%	1.4%	5.6%	2.3%	-4.5%	…	2.0%	0.1%	2.3%	-1.9%	-4.9%	1.8%
1/2/13	8.0%	2.6%	-2.0%	9.3%	6.3%	2.7%	…	8.8%	1.8%	6.6%	1.9%	2.5%	7.9%
2/1/13	4.0%	5.5%	4.7%	-6.3%	2.5%	3.9%	…	2.5%	-3.2%	3.9%	6.5%	1.2%	1.3%
3/1/13	2.2%	8.2%	11.0%	-6.0%	1.4%	5.1%	…	5.1%	7.2%	3.1%	5.5%	6.2%	4.0%
4/1/13	-1.5%	1.7%	6.3%	-2.0%	2.6%	4.6%	…	1.4%	4.6%	-2.3%	10.3%	3.8%	10.1%
5/1/13	5.7%	10.1%	8.5%	1.3%	1.4%	-5.7%	…	-2.0%	4.4%	4.4%	-10.6%	-3.2%	0.4%
6/3/13	-0.8%	-1.3%	3.4%	-3.9%	-3.7%	1.0%	…	-4.0%	4.9%	-2.1%	3.8%	-0.5%	0.1%
7/1/13	7.1%	-1.0%	2.6%	1.2%	6.2%	-0.1%	…	4.4%	10.7%	12.7%	-0.7%	4.5%	2.3%
8/1/13	-2.8%	-2.6%	-0.7%	-0.4%	-3.6%	-4.9%	…	-4.5%	-1.5%	-4.8%	-4.3%	-6.0%	-6.1%
9/3/13	5.0%	4.9%	12.3%	1.0%	0.9%	-0.1%	…	6.5%	0.2%	7.4%	-1.5%	1.3%	5.8%
10/1/13	5.3%	8.3%	10.5%	0.6%	-1.3%	4.4%	…	1.8%	-4.8%	-1.5%	9.0%	3.7%	6.2%
11/1/13	6.4%	4.8%	3.2%	1.5%	2.9%	2.3%	…	5.0%	8.7%	4.8%	-1.8%	5.4%	2.8%

12/2/13	4.9%	5.6%	1.7%	7.1%	2.0%	2.7%	…	0.3%	1.5%	2.6%	-1.0%	-2.3%	9.2%
1/2/14	-9.0%	-6.2%	-8.6%	4.0%	-11.2%	-8.8%	…	-10.8%	-4.1%	0.2%	-1.2%	-5.2%	-5.1%
2/3/14	5.6%	7.1%	3.4%	3.2%	4.1%	1.0%	…	3.1%	6.7%	3.1%	-0.9%	0.0%	10.7%
3/3/14	0.7%	-1.4%	-2.7%	2.4%	3.1%	2.0%	…	2.1%	6.3%	-0.2%	0.0%	2.9%	-0.9%
4/1/14	2.5%	-2.7%	2.8%	6.5%	5.4%	5.4%	…	6.2%	-8.9%	1.3%	-0.7%	4.2%	-0.9%
5/1/14	3.1%	4.6%	5.3%	-3.1%	-1.3%	0.3%	…	3.1%	5.9%	-1.3%	6.7%	-3.1%	5.7%
6/2/14	0.5%	3.6%	-6.1%	6.1%	6.1%	4.2%	…	1.2%	3.1%	-0.7%	-2.1%	-2.2%	2.0%
7/1/14	-1.7%	-7.2%	-5.5%	-6.9%	-1.0%	-7.5%	…	-4.9%	-0.9%	-9.3%	4.1%	-2.0%	0.2%
8/1/14	2.8%	1.7%	5.7%	7.9%	1.0%	6.0%	…	5.6%	6.7%	3.2%	-1.2%	3.2%	4.6%
9/2/14	-1.6%	-2.3%	0.5%	-9.7%	-8.1%	3.0%	…	-0.2%	-0.1%	-2.2%	0.3%	1.3%	-1.0%
10/1/14	8.2%	3.0%	-2.0%	3.1%	0.5%	-1.8%	…	7.0%	9.7%	1.3%	1.6%	-0.3%	2.6%
11/3/14	4.6%	2.7%	7.9%	-0.8%	-8.8%	7.5%	…	3.6%	3.7%	3.4%	0.7%	13.8%	1.2%
12/1/14	2.6%	0.7%	-3.3%	-9.4%	3.0%	-6.0%	…	1.9%	2.8%	4.4%	-7.8%	-1.4%	3.1%
1/2/15	-1.2%	-13.9%	11.2%	-12.7%	-9.0%	-2.5%	…	-2.9%	5.0%	-0.2%	-1.1%	-1.1%	-3.5%
2/2/15	4.5%	1.1%	4.3%	3.6%	4.9%	5.0%	…	4.4%	6.7%	6.6%	7.9%	-1.2%	13.5%
3/2/15	-2.2%	-4.0%	-0.5%	-3.5%	-1.6%	-5.7%	…	1.2%	4.4%	-3.9%	-1.7%	-1.4%	0.8%
4/1/15	-5.3%	-0.9%	-4.6%	9.0%	5.6%	0.0%	…	-6.7%	-6.0%	-3.0%	4.8%	-5.2%	3.6%
5/1/15	2.3%	2.9%	-1.3%	-1.8%	-6.5%	1.0%	…	0.0%	7.6%	3.5%	-2.0%	-4.3%	1.5%
6/1/15	-3.1%	-2.2%	-1.3%	-0.6%	-6.5%	-3.5%	…	-3.9%	1.9%	-5.5%	-5.9%	-4.6%	3.4%
7/1/15	-1.9%	-2.2%	3.9%	-6.7%	-8.6%	4.6%	…	9.3%	-0.5%	-10.1%	1.6%	1.5%	5.6%
8/3/15	-5.6%	0.9%	-9.2%	-2.8%	-7.6%	-4.4%	…	-6.4%	-4.8%	-8.4%	-1.7%	-9.9%	-16.4%
9/1/15	-0.3%	-3.4%	0.2%	-15.7%	-2.6%	2.9%	…	0.6%	0.7%	-2.9%	-5.6%	0.2%	0.3%
10/1/15	10.3%	-0.8%	12.3%	12.2%	14.2%	5.4%	…	12.6%	1.5%	10.1%	8.8%	-12.5%	10.7%
11/2/15	0.3%	-2.2%	-1.2%	-0.5%	1.7%	1.4%	…	1.5%	-4.4%	-1.8%	-3.1%	2.8%	-0.2%
12/1/15	-3.9%	-3.0%	-0.6%	-6.7%	-1.5%	0.8%	…	-1.0%	4.7%	0.0%	1.7%	4.9%	-7.0%
1/4/16	0.2%	-25.8%	-18.5%	-7.5%	-4.0%	-0.1%	…	-5.3%	-2.1%	-9.1%	9.0%	7.9%	-9.2%
2/1/16	4.5%	3.8%	-0.7%	8.4%	-2.3%	0.5%	…	0.4%	3.4%	10.4%	1.5%	0.0%	-0.3%
3/1/16	6.0%	10.0%	7.2%	12.3%	13.4%	8.1%	…	8.8%	8.3%	3.5%	6.4%	3.9%	3.9%
4/1/16	0.4%	6.8%	6.0%	2.5%	6.9%	-3.5%	…	-6.0%	2.1%	4.2%	-4.9%	-2.4%	3.9%
5/2/16	1.2%	0.5%	-5.8%	-6.9%	-0.1%	-0.4%	…	3.8%	1.5%	-3.0%	-0.1%	6.4%	-4.0%
6/1/16	4.0%	-7.4%	2.9%	4.5%	3.7%	2.4%	…	4.8%	5.9%	1.9%	9.3%	3.1%	-1.4%
7/1/16	1.8%	5.9%	2.9%	9.7%	-2.3%	-3.8%	…	-2.4%	1.4%	4.9%	0.2%	-0.1%	-1.2%
8/1/16	1.1%	1.7%	-2.4%	-1.0%	-0.8%	-0.5%	…	2.1%	-5.1%	-0.5%	-5.7%	-1.4%	-1.6%
9/1/16	-1.7%	-2.4%	1.8%	8.0%	2.3%	-1.8%	…	-3.0%	3.3%	-4.6%	-0.7%	0.9%	-1.7%
10/3/16	-6.4%	4.2%	7.8%	-5.3%	1.8%	0.2%	…	-5.7%	0.9%	0.6%	-6.6%	-3.0%	-0.2%
11/1/16	4.5%	8.1%	6.3%	13.5%	7.3%	-4.1%	…	4.7%	11.7%	5.9%	3.7%	0.6%	6.7%
12/1/16	3.9%	2.8%	3.3%	-3.0%	5.4%	2.7%	…	8.3%	1.1%	1.7%	6.7%	-1.2%	5.8%

Annex 3: Covariance matrix

	MMM	AXP	BA	CAT	CVX	KO	XOM	GE	HD	IBM	JNJ	JPM	MCD	MRK	NKE	PFE	PG	TRV	UHN	UTX	VZ	WMT	DIS
MMM	0.003	0.004	0.002	0.004	0.002	0.001	0.001	0.003	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.001	0.000	0.002
AXP	0.004	0.010	0.005	0.006	0.002	0.001	0.001	0.006	0.003	0.002	0.002	0.005	0.001	0.001	0.003	0.002	0.002	0.002	0.003	0.004	0.001	0.001	0.004
BA	0.002	0.005	0.006	0.003	0.002	0.001	0.001	0.004	0.002	0.001	0.002	0.003	0.001	0.002	0.002	0.002	0.001	0.002	0.003	0.003	0.001	0.000	0.003
CAT	0.004	0.006	0.003	0.010	0.003	0.002	0.002	0.006	0.003	0.003	0.002	0.004	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.004	0.002	0.001	0.004
CVX	0.002	0.002	0.002	0.003	0.003	0.001	0.002	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.000	0.002
KO	0.001	0.001	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001
XOM	0.001	0.001	0.001	0.002	0.002	0.001	0.002	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.002
GE	0.003	0.006	0.004	0.006	0.002	0.002	0.002	0.007	0.003	0.002	0.002	0.005	0.001	0.002	0.003	0.002	0.002	0.002	0.002	0.003	0.002	0.001	0.004
HD	0.002	0.003	0.002	0.003	0.001	0.001	0.001	0.003	0.004	0.001	0.001	0.003	0.000	0.000	0.001	0.002	0.001	0.002	0.001	0.002	0.001	0.001	0.002
IBM	0.001	0.002	0.001	0.003	0.001	0.001	0.001	0.002	0.001	0.003	0.001	0.002	0.000	0.001	0.001	0.001	0.000	0.001	0.001	0.001	0.000	0.000	0.002
JNJ	0.001	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
JPM	0.002	0.005	0.003	0.004	0.002	0.001	0.001	0.005	0.003	0.002	0.001	0.008	0.001	0.001	0.002	0.002	0.001	0.002	0.002	0.003	0.000	0.001	0.003
MCD	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.001
MRK	0.001	0.001	0.002	0.002	0.001	0.001	0.001	0.002	0.000	0.001	0.001	0.001	0.001	0.004	0.000	0.002	0.001	0.001	0.003	0.002	0.001	0.000	0.002
NKE	0.001	0.003	0.002	0.003	0.001	0.001	0.000	0.003	0.001	0.001	0.001	0.002	0.001	0.000	0.004	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
PFE	0.001	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.001	0.002	0.001	0.003	0.001	0.002	0.002	0.002	0.001	0.001	0.002
PG	0.001	0.002	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.001	0.001	0.001
TRV	0.002	0.002	0.002	0.002	0.001	0.001	0.001	0.002	0.002	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.003	0.001	0.002	0.001	0.001	0.002
UHN	0.002	0.003	0.003	0.002	0.001	0.000	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.003	0.001	0.002	0.001	0.001	0.007	0.003	0.001	0.000	0.002
UTX	0.002	0.004	0.003	0.004	0.002	0.001	0.001	0.003	0.002	0.001	0.001	0.003	0.001	0.002	0.001	0.002	0.001	0.002	0.003	0.003	0.001	0.000	0.002
VZ	0.001	0.001	0.001	0.002	0.001	0.001	0.001	0.002	0.001	0.000	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.003	0.001	0.001
WMT	0.000	0.001	0.000	0.001	0.000	0.001	0.000	0.001	0.001	0.000	0.001	0.001	0.000	0.000	0.001	0.001	0.001	0.001	0.000	0.000	0.001	0.002	0.000
DIS	0.002	0.004	0.003	0.004	0.002	0.001	0.002	0.004	0.002	0.002	0.001	0.003	0.001	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.001	0.000	0.004

Annex 4: Optimal composition of Markowitz model

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS	Sum
2/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.4%	0.0%	5.4%	23.8%	0.0%	100%
3/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.5%	0.0%	5.4%	23.7%	0.0%	100%
4/2/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.6%	0.0%	5.4%	23.6%	0.0%	100%
5/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.6%	0.0%	5.4%	23.5%	0.0%	100%
6/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.7%	0.0%	5.5%	23.4%	0.0%	100%
7/2/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.8%	0.0%	5.5%	23.2%	0.0%	100%
8/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.8%	0.0%	5.5%	23.1%	0.0%	100%
9/4/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.9%	0.0%	5.5%	23.1%	0.0%	100%
10/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.9%	0.0%	5.5%	22.8%	0.0%	100%
11/1/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.0%	0.0%	5.5%	22.8%	0.0%	100%
12/3/07	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.1%	0.0%	5.6%	22.7%	0.0%	100%
1/2/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.1%	0.0%	5.5%	22.5%	0.0%	100%
2/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.2%	0.0%	5.6%	22.3%	0.0%	100%
3/3/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.2%	0.0%	5.5%	22.3%	0.0%	100%
4/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.3%	0.0%	5.6%	22.1%	0.0%	100%
5/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.3%	0.0%	5.6%	21.9%	0.0%	100%
6/2/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.3%	0.0%	5.6%	21.8%	0.0%	100%
7/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.4%	0.0%	5.6%	21.7%	0.0%	100%
8/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.4%	0.0%	5.6%	21.5%	0.0%	100%
9/2/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.5%	0.0%	5.5%	21.4%	0.0%	100%
10/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.5%	0.0%	5.6%	21.3%	0.0%	100%
11/3/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.6%	0.0%	5.6%	21.1%	0.0%	100%
12/1/08	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.6%	0.0%	5.6%	21.0%	0.0%	100%
1/2/09	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.6%	0.0%	5.6%	20.8%	0.0%	100%
2/2/09	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.6%	0.0%	5.6%	20.8%	0.0%	100%
3/2/09	0.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.7%	0.0%	5.5%	20.7%	0.0%	100%
4/1/09	0.5%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.7%	0.0%	5.5%	20.5%	0.0%	100%
5/1/09	0.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.7%	0.0%	5.5%	20.4%	0.0%	100%
6/1/09	0.8%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.7%	0.0%	5.5%	20.3%	0.0%	100%
7/1/09	1.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.7%	0.0%	5.5%	20.2%	0.0%	100%
8/3/09	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.8%	0.0%	5.4%	20.1%	0.0%	100%
9/1/09	1.4%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.8%	0.0%	5.4%	20.0%	0.0%	100%
10/1/09	1.5%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.8%	0.0%	5.4%	19.9%	0.0%	100%
11/2/09	1.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.9%	0.0%	5.4%	19.8%	0.0%	100%
12/1/09	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.9%	0.0%	5.4%	19.7%	0.0%	100%
1/4/10	2.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.9%	0.0%	5.4%	19.6%	0.0%	100%
2/1/10	2.2%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.9%	0.0%	5.4%	19.5%	0.0%	100%
3/1/10	2.4%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.9%	0.0%	5.4%	19.4%	0.0%	100%
4/1/10	2.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.0%	0.0%	5.4%	19.3%	0.0%	100%
5/3/10	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.0%	0.0%	5.3%	19.2%	0.0%	100%

6/1/10	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.0%	0.0%	5.3%	19.1%	0.0%	100%
7/1/10	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.0%	0.0%	5.3%	19.0%	0.0%	100%
8/2/10	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.1%	0.0%	5.3%	18.8%	0.0%	100%
9/1/10	3.4%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.1%	0.0%	5.3%	18.8%	0.0%	100%
10/1/10	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.1%	0.0%	5.2%	18.7%	0.0%	100%
11/1/10	3.8%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.1%	0.0%	5.2%	18.5%	0.0%	100%
12/1/10	4.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.2%	0.0%	5.2%	18.4%	0.0%	100%
1/3/11	4.2%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.2%	0.0%	5.3%	18.3%	0.0%	100%
2/1/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.2%	0.0%	5.2%	18.2%	0.0%	100%
3/1/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.3%	0.0%	5.1%	17.8%	0.0%	100%
4/1/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.3%	0.0%	5.0%	17.5%	0.0%	100%
5/2/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.4%	0.0%	4.8%	17.2%	0.0%	100%
6/1/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.4%	0.0%	4.7%	16.9%	0.0%	100%
7/1/11	4.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.4%	0.0%	4.6%	16.6%	0.0%	100%
8/1/11	4.2%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.5%	0.0%	4.5%	16.2%	0.0%	100%
9/1/11	4.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.5%	0.0%	4.3%	15.8%	0.0%	100%
10/3/11	4.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.5%	0.0%	4.2%	15.3%	0.0%	100%
11/1/11	4.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.5%	0.0%	4.0%	14.9%	0.0%	100%
12/1/11	3.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.6%	0.0%	3.8%	14.5%	0.0%	100%
1/3/12	3.8%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.6%	0.0%	3.7%	14.1%	0.0%	100%
2/1/12	3.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.6%	0.0%	3.5%	13.7%	0.0%	100%
3/1/12	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.6%	0.0%	3.4%	13.2%	0.0%	100%
4/2/12	3.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.6%	0.0%	3.2%	12.8%	0.0%	100%
5/1/12	3.5%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.7%	0.0%	3.0%	12.4%	0.0%	100%
6/1/12	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.7%	0.0%	2.9%	12.0%	0.0%	100%
7/2/12	3.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.7%	0.0%	2.7%	11.6%	0.0%	100%
8/1/12	3.2%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.7%	0.0%	2.5%	11.1%	0.0%	100%
9/4/12	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.8%	0.0%	2.4%	10.7%	0.0%	100%
10/1/12	3.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.8%	0.0%	2.2%	10.3%	0.0%	100%
11/1/12	3.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.8%	0.0%	2.0%	9.9%	0.0%	100%
12/3/12	2.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.8%	0.0%	1.9%	9.5%	0.0%	100%
1/2/13	2.8%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.8%	0.0%	1.7%	9.0%	0.0%	100%
2/1/13	2.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.9%	0.0%	1.5%	8.6%	0.0%	100%
3/1/13	2.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.9%	0.0%	1.4%	8.2%	0.0%	100%
4/1/13	2.5%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.9%	0.0%	1.2%	7.8%	0.0%	100%
5/1/13	2.5%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.9%	0.0%	1.0%	7.4%	0.0%	100%
6/3/13	2.4%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.0%	0.0%	0.9%	7.0%	0.0%	100%
7/1/13	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.0%	0.0%	0.7%	6.5%	0.0%	100%
8/1/13	2.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.0%	0.0%	0.6%	6.1%	0.0%	100%
9/3/13	2.2%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.0%	0.0%	0.4%	5.7%	0.0%	100%
10/1/13	2.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.0%	0.0%	0.3%	5.3%	0.0%	100%
11/1/13	2.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.1%	4.8%	0.0%	100%
12/2/13	1.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	4.4%	0.0%	100%

1/2/14	1.8%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	3.9%	0.0%	100%
2/3/14	1.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	3.5%	0.0%	100%
3/3/14	1.6%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	2.9%	0.0%	100%
4/1/14	1.3%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.2%	0.0%	0.0%	2.5%	0.0%	100%
5/1/14	1.1%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	1.8%	0.0%	100%
6/2/14	0.9%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	1.1%	0.0%	100%
7/1/14	0.7%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	0.4%	0.0%	100%
8/1/14	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	3.1%	0.0%	0.0%	0.0%	0.0%	100%
9/2/14	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	2.3%	0.0%	0.0%	0.0%	0.0%	100%
10/1/14	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	1.5%	0.0%	0.0%	0.0%	0.0%	100%
11/3/14	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.7%	0.0%	0.0%	0.0%	0.0%	100%
12/1/14	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
1/2/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
2/2/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
3/2/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
4/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
5/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
6/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
7/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
8/3/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
9/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
10/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
11/2/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
12/1/15	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
1/4/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
2/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
3/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
4/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
5/2/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
6/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
7/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
8/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
9/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
10/3/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
11/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%
12/1/16	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	…	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	100%

Annex 5: Optimal composition of Black's model

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS	Sum
2/1/07	11.9%	-0.9%	0.4%	-8.6%	-5.5%	1.4%	…	-11.0%	1.6%	4.6%	8.1%	19.7%	-6.0%	1
3/1/07	15.9%	-2.9%	0.1%	-8.3%	-4.3%	1.3%	…	-9.6%	3.1%	3.1%	7.6%	18.6%	-3.0%	1
4/2/07	19.9%	-4.9%	-0.3%	-8.1%	-3.1%	1.3%	…	-8.2%	4.6%	1.5%	7.1%	17.4%	-0.1%	1
5/1/07	23.9%	-6.9%	-0.6%	-7.8%	-1.9%	1.3%	…	-6.9%	6.1%	0.0%	6.6%	16.3%	2.8%	1
6/1/07	27.9%	-8.9%	-0.9%	-7.6%	-0.7%	1.2%	…	-5.5%	7.7%	-1.6%	6.1%	15.1%	5.7%	1
7/2/07	31.9%	-10.9%	-1.3%	-7.3%	0.5%	1.2%	…	-4.1%	9.2%	-3.1%	5.7%	14.0%	8.6%	1
8/1/07	35.9%	-12.9%	-1.6%	-7.1%	1.7%	1.2%	…	-2.7%	10.7%	-4.6%	5.2%	12.8%	11.6%	1
9/4/07	39.9%	-14.9%	-1.9%	-6.8%	2.9%	1.2%	…	-1.3%	12.2%	-6.1%	4.7%	11.7%	14.5%	1
10/1/07	43.9%	-17.0%	-2.3%	-6.5%	4.0%	1.1%	…	0.0%	13.7%	-7.7%	4.3%	10.5%	17.5%	1
11/1/07	47.9%	-18.9%	-2.6%	-6.3%	5.2%	1.0%	…	1.5%	15.2%	-9.3%	3.7%	9.4%	20.4%	1
12/3/07	51.9%	-20.9%	-2.9%	-6.0%	6.4%	1.0%	…	2.8%	16.7%	-10.9%	3.3%	8.3%	23.3%	1
1/2/08	55.9%	-22.9%	-3.3%	-5.8%	7.6%	0.9%	…	4.2%	18.3%	-12.4%	2.8%	7.1%	26.2%	1
2/1/08	59.9%	-24.9%	-3.6%	-5.5%	8.8%	0.9%	…	5.6%	19.8%	-14.0%	2.3%	6.0%	29.1%	1
3/3/08	63.9%	-26.9%	-3.9%	-5.3%	10.0%	0.8%	…	7.0%	21.3%	-15.5%	1.8%	4.8%	32.1%	1
4/1/08	67.9%	-28.9%	-4.3%	-5.0%	11.2%	0.8%	…	8.4%	22.8%	-17.1%	1.3%	3.7%	35.0%	1
5/1/08	71.9%	-30.9%	-4.6%	-4.7%	12.4%	0.8%	…	9.8%	24.3%	-18.6%	0.8%	2.5%	37.9%	1
6/2/08	75.9%	-32.9%	-5.0%	-4.5%	13.6%	0.7%	…	11.2%	25.8%	-20.2%	0.4%	1.4%	40.9%	1
7/1/08	79.9%	-34.9%	-5.3%	-4.2%	14.8%	0.6%	…	12.6%	27.3%	-21.8%	-0.1%	0.2%	43.8%	1
8/1/08	83.8%	-36.9%	-5.7%	-4.0%	16.0%	0.8%	…	14.0%	28.8%	-23.2%	-0.6%	-0.9%	46.7%	1
9/2/08	87.8%	-38.9%	-6.0%	-3.7%	17.3%	0.7%	…	15.3%	30.4%	-24.8%	-1.1%	-2.1%	49.6%	1
10/1/08	91.9%	-40.9%	-6.3%	-3.5%	18.3%	0.7%	…	16.7%	31.8%	-26.3%	-1.6%	-3.3%	52.5%	1
11/3/08	95.8%	-42.9%	-6.6%	-3.2%	19.7%	0.7%	…	18.1%	33.4%	-27.9%	-2.1%	-4.4%	55.5%	1
12/1/08	99.9%	-45.0%	-7.0%	-3.0%	20.6%	0.9%	…	19.6%	34.8%	-29.3%	-2.5%	-5.6%	58.6%	1
1/2/09	104.3%	-46.9%	-7.1%	-2.4%	18.6%	0.3%	…	20.7%	36.5%	-32.3%	-3.6%	-7.3%	61.6%	1
2/2/09	108.5%	-48.9%	-7.4%	-1.8%	16.4%	-0.1%	…	21.9%	38.2%	-35.2%	-4.5%	-9.0%	64.9%	1
3/2/09	112.6%	-51.0%	-7.6%	-1.2%	14.3%	-0.5%	…	23.2%	39.9%	-38.1%	-5.5%	-10.7%	68.1%	1
4/1/09	116.9%	-53.0%	-7.7%	-0.6%	12.1%	-0.8%	…	24.4%	41.5%	-41.0%	-6.5%	-12.5%	71.3%	1
5/1/09	121.1%	-54.9%	-7.9%	0.0%	10.1%	-1.3%	…	25.6%	43.1%	-44.0%	-7.6%	-14.6%	74.3%	1
6/1/09	125.5%	-56.7%	-8.1%	0.6%	8.3%	-2.0%	…	26.8%	44.5%	-47.4%	-8.9%	-17.1%	77.2%	1
7/1/09	129.8%	-58.5%	-8.3%	1.2%	6.5%	-2.7%	…	28.1%	46.0%	-50.9%	-10.1%	-19.6%	80.1%	1
8/3/09	134.2%	-60.2%	-8.5%	1.8%	4.7%	-3.4%	…	29.3%	47.4%	-54.3%	-11.4%	-22.0%	82.9%	1
9/1/09	138.5%	-62.1%	-8.6%	2.4%	2.8%	-4.1%	…	30.5%	48.8%	-57.7%	-12.6%	-24.5%	85.9%	1
10/1/09	142.8%	-63.8%	-8.8%	3.1%	1.1%	-4.7%	…	31.8%	50.2%	-61.0%	-13.9%	-27.0%	88.7%	1
11/2/09	147.2%	-65.5%	-9.1%	3.6%	-0.7%	-5.5%	…	32.9%	51.7%	-64.3%	-15.2%	-29.5%	91.5%	1
12/1/09	151.7%	-67.3%	-9.3%	4.2%	-2.6%	-6.2%	…	34.1%	53.0%	-67.6%	-16.5%	-31.9%	94.3%	1
1/4/10	155.7%	-69.1%	-9.2%	5.0%	-4.4%	-7.0%	…	35.5%	54.5%	-71.4%	-17.6%	-34.7%	97.3%	1
2/1/10	159.9%	-71.3%	-8.7%	6.0%	-6.7%	-8.2%	…	36.5%	55.8%	-75.5%	-18.9%	-38.2%	100.7%	1
3/1/10	164.1%	-73.7%	-8.7%	6.4%	-9.0%	-10.6%	…	38.0%	57.4%	-80.3%	-20.3%	-42.0%	103.9%	1
4/1/10	168.8%	-76.4%	-9.1%	6.5%	-11.4%	-13.9%	…	39.5%	59.1%	-85.6%	-21.8%	-45.9%	106.9%	1

5/3/10	173.3%	-79.1%	-9.5%	6.6%	-13.9%	-17.3%	…	40.8%	60.8%	-90.7%	-23.2%	-49.9%	110.0%	1
6/1/10	178.0%	-81.8%	-9.9%	6.7%	-16.2%	-20.7%	…	42.3%	62.7%	-96.0%	-24.7%	-53.8%	112.9%	1
7/1/10	182.5%	-84.5%	-10.5%	6.6%	-18.9%	-24.0%	…	43.7%	64.2%	-100.0%	-26.2%	-57.8%	116.1%	1
8/2/10	186.5%	-87.4%	-12.1%	5.8%	-21.8%	-26.6%	…	44.9%	65.5%	-100.0%	-27.4%	-62.3%	119.5%	1
9/1/10	190.4%	-90.2%	-13.7%	5.0%	-24.6%	-29.2%	…	46.2%	66.7%	-100.0%	-28.6%	-66.8%	122.8%	1
10/1/10	194.3%	-93.1%	-15.2%	4.2%	-27.6%	-31.8%	…	47.5%	68.0%	-100.0%	-29.7%	-71.4%	126.3%	1
11/1/10	198.6%	-96.0%	-16.8%	3.5%	-30.7%	-34.6%	…	48.7%	69.2%	-100.0%	-30.9%	-75.9%	129.6%	1
12/1/10	202.4%	-98.9%	-18.5%	2.8%	-33.6%	-37.0%	…	50.0%	70.4%	-100.0%	-32.2%	-80.4%	133.1%	1
1/3/11	205.6%	-100.0%	-20.8%	1.5%	-36.3%	-39.9%	…	51.7%	71.7%	-100.0%	-33.4%	-85.1%	136.4%	1
2/1/11	208.3%	-100.0%	-23.7%	-0.1%	-39.1%	-43.1%	…	53.8%	72.9%	-100.0%	-34.6%	-90.1%	139.6%	1
3/1/11	211.2%	-100.0%	-26.6%	-1.7%	-41.9%	-46.1%	…	55.9%	74.1%	-100.0%	-35.7%	-95.0%	143.0%	1
4/1/11	214.0%	-100.0%	-29.7%	-3.5%	-44.5%	-49.5%	…	58.0%	75.4%	-100.0%	-37.0%	-99.9%	146.5%	1
5/2/11	217.5%	-100.0%	-32.4%	-5.2%	-47.0%	-54.2%	…	58.8%	77.1%	-100.0%	-39.0%	-100.0%	150.5%	1
6/1/11	220.9%	-100.0%	-35.3%	-6.9%	-49.5%	-59.1%	…	59.7%	78.7%	-100.0%	-41.1%	-100.0%	154.9%	1
7/1/11	224.3%	-100.0%	-38.1%	-8.7%	-51.8%	-63.8%	…	60.6%	80.4%	-100.0%	-43.3%	-100.0%	159.1%	1
8/1/11	227.7%	-100.0%	-41.0%	-10.4%	-54.3%	-68.6%	…	61.5%	82.1%	-100.0%	-45.4%	-100.0%	163.4%	1
9/1/11	231.1%	-100.0%	-43.8%	-12.2%	-56.7%	-73.5%	…	62.4%	83.7%	-100.0%	-47.5%	-100.0%	167.7%	1
10/3/11	234.5%	-100.0%	-46.8%	-13.9%	-59.1%	-78.3%	…	63.3%	85.5%	-100.0%	-49.8%	-100.0%	172.0%	1
11/1/11	238.0%	-100.0%	-49.6%	-15.7%	-61.5%	-83.2%	…	64.1%	87.1%	-100.0%	-51.8%	-100.0%	176.3%	1
12/1/11	241.3%	-100.0%	-52.5%	-17.4%	-64.0%	-88.1%	…	65.1%	88.8%	-100.0%	-54.0%	-100.0%	180.4%	1
1/3/12	244.8%	-100.0%	-55.3%	-19.2%	-66.4%	-93.1%	…	65.7%	90.4%	-100.0%	-55.9%	-100.0%	184.7%	1
2/1/12	248.4%	-100.0%	-58.2%	-21.0%	-68.9%	-97.7%	…	66.7%	92.1%	-100.0%	-58.2%	-100.0%	189.2%	1
3/1/12	253.1%	-100.0%	-60.8%	-23.0%	-72.7%	-100.0%	…	67.6%	93.5%	-100.0%	-60.1%	-100.0%	193.4%	1
4/2/12	255.1%	-100.0%	-64.2%	-24.6%	-74.2%	-100.0%	…	68.5%	96.3%	-100.0%	-64.4%	-100.0%	198.3%	1
5/1/12	258.3%	-100.0%	-67.3%	-26.4%	-76.6%	-100.0%	…	69.2%	98.7%	-100.0%	-67.7%	-100.0%	202.9%	1
6/1/12	261.9%	-100.0%	-70.3%	-28.3%	-79.3%	-100.0%	…	70.0%	101.0%	-100.0%	-70.8%	-100.0%	207.7%	1
7/2/12	267.2%	-100.0%	-73.8%	-30.4%	-83.7%	-100.0%	…	72.0%	102.3%	-100.0%	-74.2%	-100.0%	212.8%	1
8/1/12	270.1%	-100.0%	-77.4%	-32.1%	-85.3%	-100.0%	…	70.3%	104.8%	-100.0%	-78.3%	-100.0%	217.1%	1
9/4/12	273.1%	-100.0%	-82.1%	-34.1%	-88.0%	-100.0%	…	70.0%	106.4%	-100.0%	-83.7%	-100.0%	222.6%	1
10/1/12	275.7%	-100.0%	-87.2%	-35.9%	-91.0%	-100.0%	…	69.3%	108.0%	-100.0%	-90.0%	-100.0%	228.4%	1
11/1/12	278.0%	-100.0%	-92.2%	-37.7%	-94.1%	-100.0%	…	68.5%	109.6%	-100.0%	-96.2%	-100.0%	234.3%	1
12/3/12	280.6%	-100.0%	-97.4%	-39.9%	-98.0%	-100.0%	…	66.9%	111.0%	-100.0%	-100.0%	-100.0%	240.6%	1
1/2/13	280.3%	-100.0%	-100.0%	-42.9%	-100.0%	-100.0%	…	63.0%	112.4%	-100.0%	-100.0%	-100.0%	246.2%	1
2/1/13	278.4%	-100.0%	-100.0%	-46.7%	-100.0%	-100.0%	…	57.9%	113.6%	-100.0%	-100.0%	-100.0%	250.3%	1
3/1/13	274.2%	-100.0%	-100.0%	-53.2%	-100.0%	-100.0%	…	51.3%	114.3%	-100.0%	-100.0%	-100.0%	254.2%	1
4/1/13	270.1%	-100.0%	-100.0%	-59.6%	-100.0%	-100.0%	…	44.6%	115.1%	-100.0%	-100.0%	-100.0%	257.8%	1
5/1/13	266.0%	-100.0%	-100.0%	-67.1%	-100.0%	-100.0%	…	36.2%	115.7%	-100.0%	-100.0%	-100.0%	261.3%	1
6/3/13	261.8%	-100.0%	-100.0%	-77.5%	-100.0%	-100.0%	…	23.9%	116.3%	-100.0%	-100.0%	-100.0%	262.9%	1
7/1/13	257.4%	-100.0%	-100.0%	-87.9%	-100.0%	-100.0%	…	11.5%	116.9%	-100.0%	-100.0%	-100.0%	264.6%	1
8/1/13	253.0%	-100.0%	-100.0%	-98.2%	-100.0%	-100.0%	…	-0.8%	117.5%	-100.0%	-100.0%	-100.0%	266.3%	1
9/3/13	237.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-20.0%	119.1%	-100.0%	-100.0%	-100.0%	262.6%	1
10/1/13	218.5%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-40.8%	121.0%	-100.0%	-100.0%	-100.0%	257.9%	1
11/1/13	200.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-61.5%	122.8%	-100.0%	-100.0%	-100.0%	253.1%	1

12/2/13	181.6%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-82.3%	124.7%	-100.0%	-100.0%	-100.0%	248.3%	1
1/2/14	161.1%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	126.4%	-100.0%	-100.0%	-100.0%	243.1%	1
2/3/14	129.1%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	127.7%	-100.0%	-100.0%	-100.0%	235.3%	1
3/3/14	97.2%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	129.0%	-100.0%	-100.0%	-100.0%	227.4%	1
4/1/14	65.2%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	130.4%	-100.0%	-100.0%	-100.0%	219.7%	1
5/1/14	33.2%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	131.7%	-100.0%	-100.0%	-100.0%	211.8%	1
6/2/14	1.3%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	133.0%	-100.0%	-100.0%	-100.0%	203.9%	1
7/1/14	-30.7%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	134.3%	-100.0%	-100.0%	-100.0%	196.1%	1
8/1/14	-62.7%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	135.6%	-100.0%	-100.0%	-100.0%	188.2%	1
9/2/14	-94.7%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	136.9%	-100.0%	-100.0%	-100.0%	180.4%	1
10/1/14	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	134.8%	-100.0%	-100.0%	-100.0%	150.0%	1
11/3/14	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	132.1%	-100.0%	-100.0%	-100.0%	115.1%	1
12/1/14	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	129.4%	-100.0%	-100.0%	-100.0%	80.1%	1
1/2/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	126.7%	-100.0%	-100.0%	-100.0%	45.2%	1
2/2/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	124.0%	-100.0%	-100.0%	-100.0%	10.3%	1
3/2/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	121.2%	-100.0%	-100.0%	-100.0%	-24.7%	1
4/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	118.5%	-100.0%	-100.0%	-100.0%	-59.6%	1
5/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	115.8%	-100.0%	-100.0%	-100.0%	-94.6%	1
6/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	100.4%	-100.0%	-100.0%	-100.0%	-100.0%	1
7/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	82.7%	-100.0%	-100.0%	-100.0%	-100.0%	1
8/3/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	65.1%	-100.0%	-100.0%	-100.0%	-100.0%	1
9/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	47.4%	-100.0%	-100.0%	-100.0%	-100.0%	1
10/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	29.7%	-100.0%	-100.0%	-100.0%	-100.0%	1
11/2/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	12.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
12/1/15	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-5.7%	-100.0%	-100.0%	-100.0%	-100.0%	1
1/4/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-23.4%	-100.0%	-100.0%	-100.0%	-100.0%	1
2/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-41.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
3/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-58.7%	-100.0%	-100.0%	-100.0%	-100.0%	1
4/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-76.4%	-100.0%	-100.0%	-100.0%	-100.0%	1
5/2/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-94.1%	-100.0%	-100.0%	-100.0%	-100.0%	1
6/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
7/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
8/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
9/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
10/3/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
11/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1
12/1/16	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	…	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	-100.0%	1

Annex 6: Correlation matrix

	MMM	AXP	BA	CAT	CVX	KO	XOM	GE	HD	IBM	JNJ	JPM	MCD	MRK	NKE	PFE	PG	TRV	UHN	UTX	VZ	WMT	DIS
MMM	1	0.654	0.541	0.637	0.564	0.403	0.502	0.657	0.503	0.339	0.518	0.462	0.295	0.249	0.421	0.458	0.482	0.534	0.358	0.654	0.377	0.167	0.615
AXP	0.654	1	0.596	0.600	0.365	0.303	0.270	0.636	0.473	0.402	0.391	0.599	0.225	0.198	0.436	0.340	0.348	0.386	0.376	0.616	0.198	0.128	0.620
BA	0.541	0.596	1	0.407	0.396	0.338	0.356	0.604	0.369	0.328	0.476	0.444	0.369	0.482	0.369	0.484	0.303	0.461	0.508	0.681	0.268	0.107	0.649
CAT	0.637	0.600	0.407	1	0.569	0.382	0.469	0.662	0.416	0.451	0.399	0.490	0.340	0.254	0.419	0.392	0.372	0.439	0.288	0.656	0.365	0.206	0.584
CVX	0.564	0.365	0.396	0.569	1	0.403	0.800	0.482	0.280	0.391	0.385	0.318	0.440	0.338	0.185	0.330	0.325	0.469	0.212	0.498	0.430	0.083	0.561
KO	0.403	0.303	0.338	0.382	0.403	1	0.384	0.543	0.194	0.308	0.571	0.284	0.589	0.335	0.446	0.426	0.494	0.445	0.133	0.295	0.491	0.347	0.419
XOM	0.502	0.270	0.356	0.469	0.800	0.384	1	0.443	0.213	0.297	0.410	0.274	0.435	0.336	0.119	0.347	0.350	0.474	0.253	0.484	0.414	0.108	0.495
GE	0.657	0.636	0.604	0.662	0.482	0.543	0.443	1	0.500	0.414	0.536	0.616	0.406	0.304	0.539	0.531	0.560	0.532	0.348	0.646	0.370	0.259	0.690
HD	0.503	0.473	0.369	0.416	0.280	0.194	0.213	0.500	1	0.310	0.295	0.501	0.126	0.110	0.316	0.435	0.211	0.442	0.262	0.438	0.253	0.259	0.449
IBM	0.339	0.402	0.328	0.451	0.391	0.308	0.297	0.414	0.310	1	0.328	0.418	0.188	0.201	0.222	0.246	0.183	0.252	0.265	0.465	0.170	0.158	0.459
JNJ	0.518	0.391	0.476	0.399	0.385	0.571	0.410	0.536	0.295	0.328	1	0.392	0.526	0.487	0.340	0.570	0.636	0.497	0.383	0.525	0.417	0.351	0.500
JPM	0.462	0.599	0.444	0.490	0.318	0.284	0.274	0.616	0.501	0.418	0.392	1	0.287	0.173	0.404	0.503	0.321	0.494	0.296	0.578	0.104	0.226	0.550
MCD	0.295	0.225	0.369	0.340	0.440	0.589	0.435	0.406	0.126	0.188	0.526	0.287	1	0.423	0.418	0.489	0.462	0.479	0.267	0.393	0.389	0.203	0.439
MRK	0.249	0.198	0.482	0.254	0.338	0.335	0.336	0.304	0.110	0.201	0.487	0.173	0.423	1	0.101	0.523	0.290	0.420	0.493	0.409	0.339	0.157	0.385
NKE	0.421	0.436	0.369	0.419	0.185	0.446	0.119	0.539	0.316	0.222	0.340	0.404	0.418	0.101	1	0.277	0.370	0.418	0.122	0.394	0.178	0.171	0.512
PFE	0.458	0.340	0.484	0.392	0.330	0.426	0.347	0.531	0.435	0.246	0.570	0.503	0.489	0.523	0.277	1	0.453	0.544	0.498	0.531	0.368	0.349	0.549
PG	0.482	0.348	0.303	0.372	0.325	0.494	0.350	0.560	0.211	0.183	0.636	0.321	0.462	0.290	0.370	0.453	1	0.442	0.276	0.419	0.293	0.303	0.412
TRV	0.534	0.386	0.461	0.439	0.469	0.445	0.474	0.532	0.442	0.252	0.497	0.494	0.479	0.420	0.418	0.544	0.442	1	0.324	0.526	0.402	0.374	0.512
UHN	0.358	0.376	0.508	0.288	0.212	0.133	0.253	0.348	0.262	0.265	0.383	0.296	0.267	0.493	0.122	0.498	0.276	0.324	1	0.544	0.213	0.045	0.453
UTX	0.654	0.616	0.681	0.656	0.498	0.295	0.484	0.646	0.438	0.465	0.525	0.578	0.393	0.409	0.394	0.531	0.419	0.526	0.544	1	0.251	0.162	0.659
VZ	0.377	0.198	0.268	0.365	0.430	0.491	0.414	0.370	0.253	0.170	0.417	0.104	0.389	0.339	0.178	0.368	0.293	0.402	0.213	0.251	1	0.268	0.332
WMT	0.167	0.128	0.107	0.206	0.083	0.347	0.108	0.259	0.259	0.158	0.351	0.226	0.203	0.157	0.171	0.349	0.303	0.374	0.045	0.162	0.268	1	0.153
DIS	0.615	0.620	0.649	0.584	0.561	0.419	0.495	0.690	0.449	0.459	0.500	0.550	0.439	0.385	0.512	0.549	0.412	0.512	0.453	0.659	0.332	0.153	1

Annex 7: Optimal composition of Mean-VaR model ($\alpha = 0.01$)

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS	Sum
2/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	23.1%	1.0%	100.0%
3/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	23.1%	1.0%	100.0%
4/2/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	23.0%	1.0%	100.0%
5/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.9%	1.0%	100.0%
6/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.8%	1.0%	100.0%
7/2/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.7%	1.0%	100.0%
8/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.6%	1.0%	100.0%
9/4/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.4%	1.0%	100.0%
10/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.3%	1.0%	100.0%
11/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.2%	1.0%	100.0%
12/3/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.1%	1.0%	100.0%
1/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	22.1%	1.0%	100.0%
2/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.9%	1.0%	100.0%
3/3/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.9%	1.0%	100.0%
4/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	21.7%	1.0%	100.0%
5/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	21.7%	1.0%	100.0%
6/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.6%	1.0%	100.0%
7/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.5%	1.0%	100.0%
8/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.3%	1.0%	100.0%
9/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	21.2%	1.0%	100.0%
10/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	21.1%	1.0%	100.0%
11/3/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	21.0%	1.0%	100.0%
12/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.9%	1.0%	100.0%
1/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.8%	1.0%	100.0%
2/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.7%	1.0%	100.0%
3/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.7%	1.0%	100.0%
4/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.5%	1.0%	100.0%
5/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.4%	1.0%	100.0%
6/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.3%	1.0%	100.0%
7/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.2%	1.0%	100.0%
8/3/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.3%	20.1%	1.0%	100.0%
9/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.3%	19.9%	1.0%	100.0%
10/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	19.9%	1.0%	100.0%
11/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	19.8%	1.0%	100.0%
12/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	19.7%	1.0%	100.0%
1/4/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.0%	19.5%	1.0%	100.0%
2/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.9%	19.3%	1.0%	100.0%
3/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.9%	19.0%	1.0%	100.0%
4/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.7%	18.7%	1.0%	100.0%

5/3/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.5%	18.4%	1.0%	100.0%
6/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.4%	18.1%	1.0%	100.0%
7/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.3%	17.8%	1.0%	100.0%
8/2/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.2%	17.5%	1.0%	100.0%
9/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.0%	17.2%	1.0%	100.0%
10/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.9%	16.9%	1.0%	100.0%
11/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.8%	16.6%	1.0%	100.0%
12/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.7%	16.3%	1.0%	100.0%
1/3/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.5%	16.0%	1.0%	100.0%
2/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.4%	15.7%	1.0%	100.0%
3/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.3%	15.4%	1.0%	100.0%
4/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.2%	15.1%	1.0%	100.0%
5/2/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.0%	14.8%	1.0%	100.0%
6/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.9%	14.5%	1.0%	100.0%
7/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.8%	14.3%	1.0%	100.0%
8/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.6%	14.0%	1.0%	100.0%
9/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.5%	13.7%	1.0%	100.0%
10/3/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.4%	13.4%	1.0%	100.0%
11/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.3%	13.1%	1.0%	100.0%
12/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.1%	12.8%	1.0%	100.0%
1/3/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.0%	12.5%	1.0%	100.0%
2/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.9%	12.2%	1.0%	100.0%
3/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.8%	11.9%	1.0%	100.0%
4/2/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.6%	11.6%	1.0%	100.0%
5/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.5%	11.3%	1.0%	100.0%
6/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.4%	11.0%	1.0%	100.0%
7/2/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.3%	10.7%	1.0%	100.0%
8/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.1%	10.4%	1.0%	100.0%
9/4/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	10.1%	1.0%	100.0%
10/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.7%	1.0%	100.0%
11/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.4%	1.0%	100.0%
12/3/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.0%	1.0%	100.0%
1/2/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.7%	1.0%	100.0%
2/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.3%	1.0%	100.0%
3/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.0%	1.0%	100.0%
4/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	7.5%	1.0%	100.0%
5/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	7.0%	1.0%	100.0%
6/3/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	6.5%	1.0%	100.0%
7/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	6.0%	1.0%	100.0%
8/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	5.4%	1.0%	100.0%
9/3/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	4.9%	1.0%	100.0%
10/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	4.4%	1.0%	100.0%
11/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	3.8%	1.0%	100.0%

12/2/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	3.3%	1.0%	100.0%
1/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	2.8%	1.0%	100.0%
2/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	2.2%	1.0%	100.0%
3/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.7%	1.0%	100.0%
4/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.2%	1.0%	100.0%
5/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
1/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
2/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
3/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
4/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
5/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/3/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
1/4/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
2/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
3/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
4/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
5/2/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/3/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%

Annex 8: Optimal composition of Mean-VaR model ($\alpha = 0.15$)

Date	MMM	AXP	BA	CAT	CVX	KO	…	TRV	UHN	UTX	VZ	WMT	DIS	Sum
2/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.7%	1.0%	100.0%
3/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.7%	1.0%	100.0%
4/2/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.6%	1.0%	100.0%
5/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.5%	1.0%	100.0%
6/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.4%	1.0%	100.0%
7/2/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.4%	1.0%	100.0%
8/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.3%	1.0%	100.0%
9/4/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.3%	20.1%	1.0%	100.0%
10/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.1%	1.0%	100.0%
11/1/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	20.0%	1.0%	100.0%
12/3/07	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	19.9%	1.0%	100.0%
1/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.2%	19.8%	1.0%	100.0%
2/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	19.8%	1.0%	100.0%
3/3/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	19.7%	1.0%	100.0%
4/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.1%	19.5%	1.0%	100.0%
5/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	5.0%	19.4%	1.0%	100.0%
6/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.9%	19.2%	1.0%	100.0%
7/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.7%	18.9%	1.0%	100.0%
8/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.6%	18.7%	1.0%	100.0%
9/2/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.6%	18.5%	1.0%	100.0%
10/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.5%	18.2%	1.0%	100.0%
11/3/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.3%	17.9%	1.0%	100.0%
12/1/08	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.2%	17.7%	1.0%	100.0%
1/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.1%	17.5%	1.0%	100.0%
2/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.0%	17.2%	1.0%	100.0%
3/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	4.0%	17.0%	1.0%	100.0%
4/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.9%	16.8%	1.0%	100.0%
5/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.8%	16.5%	1.0%	100.0%
6/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.6%	16.3%	1.0%	100.0%
7/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.5%	16.1%	1.0%	100.0%
8/3/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.4%	15.8%	1.0%	100.0%
9/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.3%	15.6%	1.0%	100.0%
10/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.3%	15.3%	1.0%	100.0%
11/2/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.1%	15.1%	1.0%	100.0%
12/1/09	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	3.0%	14.9%	1.0%	100.0%
1/4/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.9%	14.6%	1.0%	100.0%
2/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.8%	14.4%	1.0%	100.0%
3/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.7%	14.2%	1.0%	100.0%
4/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.6%	13.9%	1.0%	100.0%

5/3/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.5%	13.7%	1.0%	100.0%
6/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.4%	13.4%	1.0%	100.0%
7/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.3%	13.2%	1.0%	100.0%
8/2/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.2%	13.0%	1.0%	100.0%
9/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.1%	12.7%	1.0%	100.0%
10/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	2.0%	12.5%	1.0%	100.0%
11/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.9%	12.3%	1.0%	100.0%
12/1/10	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.8%	12.0%	1.0%	100.0%
1/3/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.7%	11.8%	1.0%	100.0%
2/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.6%	11.6%	1.0%	100.0%
3/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.5%	11.3%	1.0%	100.0%
4/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.4%	11.1%	1.0%	100.0%
5/2/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.3%	10.8%	1.0%	100.0%
6/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.2%	10.6%	1.0%	100.0%
7/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.2%	10.3%	1.0%	100.0%
8/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	10.1%	1.0%	100.0%
9/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.9%	1.0%	100.0%
10/3/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.6%	1.0%	100.0%
11/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.3%	1.0%	100.0%
12/1/11	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	9.0%	1.0%	100.0%
1/3/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.7%	1.0%	100.0%
2/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.5%	1.0%	100.0%
3/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	8.2%	1.0%	100.0%
4/2/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	7.9%	1.0%	100.0%
5/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	7.5%	1.0%	100.0%
6/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	7.1%	1.0%	100.0%
7/2/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	6.7%	1.0%	100.0%
8/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	6.2%	1.0%	100.0%
9/4/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	5.8%	1.0%	100.0%
10/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	5.4%	1.0%	100.0%
11/1/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	5.0%	1.0%	100.0%
12/3/12	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	4.5%	1.0%	100.0%
1/2/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	4.1%	1.0%	100.0%
2/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	3.7%	1.0%	100.0%
3/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	3.3%	1.0%	100.0%
4/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	2.8%	1.0%	100.0%
5/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	2.4%	1.0%	100.0%
6/3/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	2.0%	1.0%	100.0%
7/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.6%	1.0%	100.0%
8/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.2%	1.0%	100.0%
9/3/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/1/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%

12/2/13	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
1/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
2/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
3/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
4/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
5/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/2/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/3/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/14	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
1/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
2/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
3/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
4/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
5/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/3/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/2/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/15	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
1/4/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
2/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
3/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
4/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
5/2/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
6/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
7/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
8/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
9/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
10/3/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
11/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%
12/1/16	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	…	1.0%	1.0%	1.0%	1.0%	1.0%	1.0%	100.0%